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On Homogeneous Structures and the Symmetrical Partitioning of them, with application to Crystals.

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AT the outset we have to consider what constitutes homogeneity of structure.

First, it may be noted that it is not to be confounded with ***structureless homogeneity***.

The completest homogeneity conceivable is presented when at *every geometrical point* in space the qualities are the same; but when this is the case, structure is absent, and no property can be displayed depending on differences of any kind which enable us to distinguish one point from another, however infinitesimal the distance separating the points compared. Homogeneity of structure consists, not in the same qualities being presented at every geometrical point of a body, but in the similar repetition of the same qualities or forms throughout it.

In Thomson and Tait's treatise on Natural Philosophy we read, "A body is called homogeneous when any two equal, similar parts of it, with corresponding lines parallel and turned towards the same parts, are undistinguishable from one another by any difference in quality;" but this definition, when applied to a homogeneous structure, cannot be other than empirical and not mathematically precise, its truth depending on the tests applied not being refined enough to detect ultimate discontinuity of constitution of the structure.[1] It may be said, as applied to such a structure, to describe an apparent property rather than to lay down a geometrical definition. The writer has endeavoured to frame a definition of homogeneity of structure which shall be mathematically exact, and free from any needless limitation, and recently propounded the following:[2] "A homogeneous structure is one every point within which, if we regard the structure as without boundaries, has corresponding to it an infinitude of other points whose situations in the structure are precisely similar, so that all of the infinite number of geometrical point-systems respectively obtained by taking all similarly situated points are regular infinite point-systems, defined by Sohncke as systems of points such that the arrangement about any one of these points of the rest of the points of the system is the same as it is about any other of them."[3]

It may be remarked in this connection that the conception of a system of points or atoms which repeats itself in the way indicated presented itself independently to both Sohncke and Wiener.[4]

Here are some models which I have constructed to show some of the least complicated types of homogeneous structure of the cubic system, and which will, I hope, make my position clearer; and in reference to them I may say that my mind has been exercised in attempts to portray the nature of the repetition which constitutes homogeneity of structure in the most general way possible, and especially to avoid anything which seems to imply atomicity, proof of the existence of which must, it would appear, be sought elsewhere than in the geometry of crystals, *i.e.* of homogeneous structures.

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[1] Fedorow cites some evidences of discontinuity of structure of crystals. See *Theorie der Krystallstruktur* in *Zeitschr. für Kryst. &c.* XXV. p. 116. Comp. Thomson and Tait, II. p. 216.

[2] *Zeitschr. für Kryst. &c.* XXIII. p. 1.

[3] Sohncke's *Entwickelung einer Theorie der Krystallstruktur*, p. 28. Those kinds of repetition in space the repeating parts of which have some dimension infinite, are not included in this inquiry.

[4] Comp. Wiener's *Grundzüge der Weltordnung*. Leipzig, 1869. Atomlehre, p. 82.

With this purpose before me, I have, in devising my models, avoided the use of particles or spheres, and have placed a quite irregular body, a hand,[1] in the positions appropriate for showing the kind of repetition characteristic of a given type.

This model (a cubic element of which is shown diagrammatically in fig. 1) gives a certain very regular type of cubic symmetry, that which is marked $7a_1$ in the lists of homogeneous structures to be found in the memoir above referred to.[2] It belongs to Class 31 (dodecahedral hemihedry) in Sohncke's list of 32 classes of crystal symmetry.[3] The structure contains left hands as well as right hands, because it is one of those that are identical with their own mirror-images, and consequently made up of parts enantiomorphically related. It is formed of a number of similar groups of the kind shown, the cubes containing which are fitted together to fill space symmetrically.

In the actual model each such cubic element is composed of eight *cubelets*, the edges of which are formed of strips of wood meeting at the corners of the cubelet, and the latter having also one of its four diagonals occupied by a strip of wood on which three hands are hung symmetrically.

Thus each cubelet contains three of the 24 hands found in a single element, four alternate cubelets being similarly occupied by right hands, the remaining four by left hands. The entire structure is therefore built up of cubelets of two kinds, with their respective triads of hands.

The eight single diagonals of the eight cubelets of an element meet in a point at its centre.

The cubic partitioning thus indicated is, however, not essential, and is to be regarded as mere scaffolding, and as having fulfilled its object in keeping the hands in their places, and in enabling us better to perceive the nature of their arrangement; *indeed, the type of homogeneity is expressed in a more general manner when there is no partitioning at all.*[4]

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[1] Comp. Fedorow, *Zeitschr. für Kryst.* XXV. p. 115. The employment of hands was suggested to me by Prof. Miers; its advantages are that the use of so familiar an object greatly facilitates a clear perception of the nature of the arrangement, and that the shape is so exceptional that no one can be led to imagine it to be a necessary feature of any type.

[2] *Zeitschr. für Kryst.* XXIII. p. 44.

[3] *Zeitschr. für Kryst.* XX. p. 467.

[4] This is true of all the types. The same homogeneous structure can, in most cases, be partitioned in a manner compatible with the preservation of its homogeneity and symmetry *in an infinite number of ways.* The cubic partitioning resorted to in the case under consideration is a very arbitrary one, and a structure of the given type need not have any characteristics corresponding to the *exceptional* symmetry which this kind of partitioning produces.

As we shall see presently, there are cases of homogeneous structure in which *no*

In the case before us, the edges of the cubelets are digonal axes of rotation of the structure, *i.e.* when the structure is turned through 180° about one of them, all its parts in the new position coincide with all its parts in the old. The positions of the planes of symmetry of the structure, as well as those of the various axes, are readily traceable when a number of composite cubic groups of the kind described are stacked together.

The right hands are distributed homogeneously, and so are the left hands, and, in obedience to the definition above given, any set of similar points, *e.g.* the tips of the thumbs, or the tips of the first fingers of either right hands or left hands, form a Sohnckian point-system. If we take corresponding points of both right and left hands, we obtain the double point-systems of Fedorow. The single point-systems traceable in this type are those described by Sohncke as No. 54; it is easy to see that all the points in any one of them have the fundamental property of a regular infinite point-system; they bear identically the same relation to the infinite structure considered without reference to the horizon, or the north pole, or indeed anything other than the homogeneous structure itself.

One of the simplest possible examples of this type of structure will be obtained if we make a stack of similar bricks in the following way; and to make the description simpler, we will employ bricks whose width and thickness are respectively half and quarter of the length.

Stand two bricks on end opposite and parallel, with a space between them of the width of a brick, the two thicknesses and the space together thus making up the length of a brick.

In the space lay a brick flat, so as to project the thickness of a brick on each side beyond the two first placed.

On the projecting ends of this brick stand two bricks on edge, with their overhanging ends even with the faces of the two first placed.

Finally, place a brick flat directly over the middle one, with its ends resting on the two last placed.

The outline of the pile thus formed will be a cube, except that a cube-shaped vacancy will be left at each of the eight corners. There will also be a cubic cell left in the middle.

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kind of partitioning *into single units* (*i.e.* not of two kinds which are enantiomorphs) is possible *which does not impair the symmetry and alter the type of homogeneity.* (See p. 132 and $2b_1$ and $6b_2$ in table on p. 135).

Some suggestions for the classification of the different kinds of *symmetrical* partitioning possible will be made presently.

Stack together a number of cubic elements of this kind similarly orientated, with the cubes of succeeding layers directly over one another. The homogeneous structure obtained is an example of the type above given.

As the outside faces of the bricks of each cubic element coincide with the faces of those in contact with it belonging to the adjoining elements, the stack may be regarded as composed of pairs of bricks, each pair consisting of two bricks placed face to face. No change of type will therefore be involved if, instead of pairs of bricks, single bricks of double the size are used, *i.e.* such as have width and thickness equal, and each equivalent to half the length. Each double brick will then fill the space previously occupied by two of the thinner ones.

Other types of homogeneous structure also of the cubic system are represented by these remaining models, composed of hands appropriately arranged.

Thus we have :—

Type 7, a representative of Class 32, the tetartohedral cubic (an element of this structure is represented by fig. 2). Of the eight cubelets which compose the element, four are those containing right hands (or left hands), and the remaining four are empty.

Type $7b_1$, a representative of Class 30, tetrahedral hemihedry (an element is represented by fig. 3). Of the eight cubelets composing the element, four contain both right and left hands symmetrically placed, and four are empty.

Type 12, a representative of Class 29, the gyrohedral cubic (an element is represented by fig. 4). The eight cubelets composing the element, all contain right hands or all left hands.

Type $12a_1$, a representative of Class 28, the holohedral cubic (an element is represented by fig. 5). The eight cubelets comprising the element all contain both right hands and left hands.

The simplest possible example of a structure of the last-named type is presented by a cubic partitioning of space whose cells *are empty*. We have too an extremely simple example of this type in a cubic Raumgitter.

Here, too, are some types of structure which are most conveniently constructed by packing close together cells which are respectively dodecahedra and cubo-octahedra, but as before the cellular construction is to be regarded as mere scaffolding.

Type 6. This, like type 7, is a representative of Class 32, the tetartohedral cubic (an element contained in a dodecahedral cell is represented by fig. 6). This type can also be built up of cubelets, but there will be a larger number of empty ones than in the former cases.

Type 10. This also, like type 7, is a representative of Class 32, the tetartohedral cubic (an element contained in a cubo-octahedral cell is represented by fig. 7).

If cubelets with a single diagonal are employed to build up this type, they must each contain three right hands or three left hands appropriately placed, cubelets which have a common diagonal being everywhere found similarly orientated.

The element contained in the dodecahedral cell of type 6 is precisely of the same kind as that contained in the cubo-octahedral cell of type 10. Similar points form, in both cases, 12-punkters of Sohncke.

Cubelets with a single diagonal, of the kinds above described, can also be employed to construct the less-regular types of structure of the cubic system.

In the memoir referred to[1] I have made an investigation of all the possible types of structure which fulfil the definition given above. The result shows that all homogeneous structures have a general symmetry which is strictly paralleled by the symmetry of crystals. And, while it is true that the definition laid down does not express any limitation regarding the relative orientations of the similar parts[2] whose likeness constitutes the homogeneity, much less require the orientations to be identically the same, as is the case with some of the older definitions of homogeneity, we discover that similar portions of a homogeneous structure always have a definite number of symmetrically related orientations, depending on which among the 32 classes of crystal symmetry the structure selected belongs to.

And this number is that of the different orientations of the ideal crystal form which can be made and give coincidence of outline. It is therefore in some but 1; in others 2, 3, 4, 6, 8, 12, or 24, as the case may be.

The relation of the orientations of similar parts just referred to *should not, therefore, be made a matter of definition, but of proof.* If a likeness of parts obeying the definition which I have given existed which did not give the symmetrical relative orientations referred to, it would notwithstanding be an example of homogeneity of structure; but, in fact, as I have just intimated, there is no exception, the only case where there is homogeneity without symmetrically related orientation of similar parts being that of structureless homogeneity.

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[1] As corrected in some few particulars in an article in the same journal, Vol. XXV. p. 86.

[2] By similar parts is meant parts similarly related to the whole.

There is a more uniform quite universal property of homogeneous structures, which may be expressed in much the same words as those of Thomson and Tait's definition quoted above.

Thus, such a structure can always be regarded as made up of equal, similar, sameways-orientated parts, which are undistinguishable from one another by any difference in quality.[1]

Each of the parts thus related will consist of an integral number, large or small, of space-units[2] of the structure of some kind or other. In the case of a structure with anorthic symmetry, it may comprise but one space-unit only. The distances between corresponding points of two such adjoining parts will be those between the corresponding points of some two contiguous or more or less widely separated space-units *whose orientation is the same.*

The outlines of the parts thus related will generally not constitute a partitioning which can represent any possible natural partitioning of crystals into chemical molecules, or into crystal molecules (if there be such entities distinct from chemical molecules); for, except in a few particular types of cases, the partitioning arrived at in this way will be quite arbitrary, and be incompatible with some of the coincidence-movements (Deckbewegungen[3]) of the structure, and therefore destructive of a portion of its symmetry, and will effect a change of its type.

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[1] Fedorow says, "Das Resultat der Beobachtungen kann auch dahin gedeutet werden, dass die krystallinisch-homogene Substanz aus gleichen und gleich-orientirten (d. h. sämmtlich in paralleler Lage geordneten) Theilchen besteht, welche zusammen genommen den Raum lückenlos ausfüllen."—*Zeitschr. für Kryst.* XXV. p. 117.

[2] The three necessary properties of a space-unit are—(1) it is continuous; (2) it contains every kind of point of the structure (*i.e.* every kind of standpoint from which the structure can be regarded); (3) all the points in it are differently related to the structure as a whole. See *Zeitschr. für Kryst.* XXIII. p. 38.

[3] The following is Sohncke's definition of a coincidence-movement (*Deckbewegung*):—

Suppose a regular infinite point-system (see p. 120 above) to be made rigid, and moved bodily out of its original position. The positions originally occupied by the points of such a system then collectively furnish a point-system which is congruent with the moved system, and which, to distinguish it from the *moveable system*, may be called the *fixed* system.

If now any point of the moveable system be selected, and placed to coincide with any point whatever of those of the fixed system, it is, owing to the congruence of all *Linienbündel*, always possible to bring the two systems to coincidence.

The movement which the moveable system executes in order to pass from one position of coincidence with the fixed system to some other position of coincidence, is called a *Deckbewegung*.

Such Deckbewegungen can be partly parallel translations, partly rotations or screw movements about certain axes definitely placed in the fixed system.

The Deckbewegungen possessed by any particular system will depend on the properties of the system, and indeed the different kinds of point-systems may be

The property referred to is therefore of no great importance as regards the ultimate structure of crystals, although it may possibly aid our conception of the connection between homogeneity of structure and some physical properties.[1]

The definition above given is, as we have seen, not limited in its application to point-systems or assemblages of particles; it may be obeyed by any kind of structure, whether material or merely geometrical, whether filling space or continuously ramifying through it, or distributed through it in discrete patches. It may, too, be obeyed by structures whose parts are in motion, provided the similarity extends to the movement of similar parts; but the similar movements need not be simultaneous.[2]

The number of different types of symmetrical arrangement presented by homogeneous structures is 230;[3] and these types, as we have said, fall into 32 classes, whose general symmetry is of the 32 kinds proper to crystals.

Such an array of possible types may well alarm the crystallographic student, and lead him either to quit the subject of homogeneous symmetry in despair, or to beat about for some pretext which shall justify the exclusion of some of these types, and bring the number available for crystals down to a smaller figure.

I deprecate the latter course, unless the grounds for it are fully adequate; otherwise the very types we exclude may turn out after all to be those which are most suggestive in connection with crystal properties.

The course which seems to me more scientific, and I may add more hopeful, is to seek to become acquainted with the principal features of homogeneous structures generally, *and especially with the numerous geometrical properties which they possess in common.*

The models which I have made are, I think, calculated to give some material help in this direction; even a cursory glance at them reveals such intimate relations and similarities between types which eminent authorities say we may reject for crystallographic purposes, and others

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distinguished according to the Deckbewegungen which are proper to them, so that the different sets of Deckbewegungen presented serve as the ground of the classification of the regular point-systems. (See *Entwickelung einer Theorie, &c.* p. 28. Compare Bravais, "Sur les Systèmes formés par des points distribués régulièrement," *Journ. de l'Ecole Polytechnique, Cahier*, XXXIII. p. 57.)

[1] See below, p. 127.

[2] They may resemble the rhythmically related movements of *combined figure skating.*

[3] This number agrees with the total number of single and double point-systems as ascertained by Fedorow and Schönflies independently.

which they say we may retain, as will at least make us examine very closely, and indeed suspiciously, the grounds on which they consider themselves authorised to discriminate.

I allude more particularly to Fedorow's attempt to select from among the types of homogeneous structure those which are possible for crystals, and to determine the shapes of their ultimate units,[1] in reference to which I cannot but express my regret that so eminent an authority on crystal structure should give the weight of his name to what appear to me to be *a number of wholly untenable limitations*.

Fedorow says that the result of observations of the properties of crystals may be summed up in the proposition that the homogeneous crystal-substance consists of similar, sameways-orientated parts which, taken together, fill space without interstices,[2] and proceeds to argue that some of the 230 types of homogeneous structure are incapable of being thus divided, and must therefore be regarded as impossible for crystals.[3] But is this so? It has already been pointed out above that every homogeneous structure, *without exception*, can be regarded as made up of equal similar, sameways-orientated parts, which are undistinguishable from one another by any difference in quality, and there does not appear to be any reason why a structure possessing this property should not fulfil the experimental requirements of crystal-structure *even in cases where the sameways-orientated parts are merely geometrical ones, the existence of whose boundaries lowers the symmetry of the structure as a whole and alters its type*, and which do not, therefore, represent any separable units (molecules or other).[4]

We have indeed to ask ourselves continually in connection with this and the other contentions of Fedorow, contained in the same treatise, what significance he attaches to his partitioning of a crystal structure? Is it suggested that when a crystal dissolves, its parts will under any circumstances retain shapes which, if fitted together, fill space, or, to put it more precisely, that the solid angles and edges of the cells are none of them vacuous or structureless in the unbroken structure, but always occupied by matter which, when the structure breaks up, can retain its situation relatively to other matter belonging to the same cell? Such a

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1 Fedorow's *Theorie der Krystallstruktur. Mögliche Strukturarten. Zeitschr. für Kryst.* XXV. p. 113.

2 *Zeitschr. für Kryst.* XXV. p. 117.

3 *Ibid.* See especially p. 218.

Cases of this kind are presented by types 3 and 4 in my list. *Zeitschr. für Kryst.* XXIII. p. 11. Compare below, pp. 128 and 135.

proposition is so unlikely that to state it is almost to refute it. Certainly we cannot, until evidence is forthcoming, assume that there is any truth in it.

But if, as is then almost certain, the partitioning into cells of a definite outline capable of filling space is, in the case of crystals, *a mere geometrical fiction*, what stress can be laid on the relative inclinations of the plane-faces of the cells,[1] or how can re-entrant angles be argued to be unlikely, or indeed what significance is there in the cell-walls having plane-faces at all?

If a particular partitioning has no strict counterpart in a crystal, but simply expresses the nature of its divisibility into units or molecules in a general manner, the presence of a re-entrant angle in a cell wall would merely represent that the outline or boundary of one composite unit or molecule fits into that of an adjoining one like the parts of a carpenter's "dovetail," or like a tenon into a mortise, a conception by no means startling or improbable when taken in as general a sense as possible.

We cannot, therefore, concur with Fedorow in his rejection of those asymmorphous types which cannot be partitioned into similar cells with similar contents, whose *outline*[2] *displays the complete symmetry of the entire structure*, or accept his reasons for regarding as unlikely for crystals cases where the partitioning into plane-faced cells gives either (*a*) abnormal (anomale) parallelohedra, *i.e.* parallelohedra whose form is such that no kind of homogeneous deformation reduces them to the most symmetrical forms of crystal symmetry,[3] or (*b*) re-entrant angles between the plane walls of a cell,[4] or (*c*) cells whose external boundaries are similarly orientated, while the contents of the cells are not.[5]

It is true that Fedorow's classification of homogeneous structures under three heads—symmorphous, hemi-symmorphous, and asymmorphous systems—marks geometrical differences whose nature is thus

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1 Fedorow distinguishes "normale" and "anomale" parallelohedra (see below).

2 Comp. Fedorow, *Zeitschr. &c.* XXV. Note*, p. 147.

3 *Ib.* p. 132.

4 *Ib.* p. 133.

5 Fedorow calls these "extraordinär," those whose contents and outlines are alike sameways-orientated "ordinär" (see p. 135).

The suggestion made by this author (p. 146) that cases of the "extraordinären" kind are unlikely, because it is inconceivable why the contents of the cells should be differently orientated, does not help his contention, because, if there is this difficulty in accounting for various orientations of the composite elements of a mass, *there is also the same kind of difficulty* in accounting for the similar parts of any such element being put together with their orientations various, and the latter is an essential condition in most homogeneous structures.

appropriately indicated, and that his other distinctions are also based on geometrical facts; but this does not strengthen his position so long as he fails to bring forward any evidence which is even plausible that any one of the classes of a highly specialised nature which he discriminates is more likely than another to be present in crystals.[1]

Instead of dwelling on differences between the various kinds of plane-faced cells—an investigation which may or may not ultimately prove to be of any importance—we ought, as it seems to me, to strive after some altogether broader treatment of the subject, which will classify the various kinds of partitioning possible in a perfectly general manner, without arbitrarily rejecting any.

Reasons are not wanting for undertaking an investigation of this sort.

Thus the fact that bodies which form crystals are capable of passing to a liquid state and back again to the crystallised condition *without breaking up into their constituents*, and the evidence we often have of the survival in the liquid of some portion of the symmetry of arrangement of parts prevailing in the solid, *notably in the cases of those bodies which rotate the plane of polarisation*, lead us to conclude that crystallised matter can be broken up into particles or units which are all alike, and each of which has parts or properties that have some definite arrangement relatively to one another.[2]

In some cases there is evidence of the survival in the dissolved crystal of two kinds of arrangement which are enantiomorphous, *e.g.* racemic compounds can mechanically or otherwise be shown to be composed of two isomers which are respectively right handed and left handed, but otherwise alike.

In the light of such facts as these the subject appears to be interesting, and likely to prove at least suggestive; the remainder of this paper is, therefore, devoted to an attempt to indicate broadly the lines on which it may be dealt with.

The symmetrical partitioning of homogeneous structures.

The existence of a homogeneous structure whose stability is such that it withstands internal or external forces, implies the existence of ties or bonds of some sort which keep it together, and preserve, or continually

[1] There does not indeed appear to be any evidence that the divisions of Fedorow's classification are at all traceable by the experimental facts at present known to us regarding crystals.

[2] This is true, whether there is any such thing as a crystal molecule distinct from a chemical molecule or not. There does not, it may be remarked, appear to be any adequate evidence of such a distinction.

restore its form, and the arrangement of these ties will conform to the definition of homogeneity. And in the case of a structure identical with its own mirror-image, the system they form will also possess the latter property.

Any *uniformly applied* disintegrating force will, as a consequence of the symmetry, break the ties, whatever they may be, in a uniform manner, so that the loosing of any given tie will be accompanied by the loosing of all ties similar to it throughout the structure; and if the disintegration leaves some ties intact, these also will be such as were similarly placed in the undissolved structure, *i.e.* the survival of any given tie will involve the survival of all ties similar to it in nature and situation. The application of a uniformly applied disintegrating force will not, therefore, produce the kind of partitioning above referred to, which is incompatible with the coincident-movements (Deckbewegungen)[1] of the structure.[2]

1. If the breaking of the ties does not bring about the absolute fluidity of any portions of the structure, but merely its separation into parts of relatively small magnitude, whose united bulk is that of the space originally occupied by the structure,[3] the latter will come to consist of a collection of fragments, either of one kind only, or of a definite number and variety of different kinds, continually repeated throughout space, and *of such form or forms as will, when properly fitted together, fill space without interstices.*

2. If, however, any parts of the structure become absolutely fluid or are vacuous, while other parts remain rigid, *i.e.* contain undissolved ties, the effect of the uniform loosing of some of the ties accompanied by disturbance will be to reduce the mass to a collection of rigid fragments, either all of the same pattern, or of a limited number of patterns, but not capable, as in the former case, of completely filling space; and they will be interspersed through the structureless fluid, that is if such a fluid, and not a vacuum, occupied in the dissolving structure the rest of space.[4]

Now if we confine our attention to simple structures only, *i.e.* to those whose ultimate fragments or particles, when liquefaction ensues, are all

[1] See Note 3, p. 125. [2] See page 125.

[3] Expansion or contraction of the parts of the structure is neglected.

[4] If the ideas generally held respecting the nature of matter are substantially correct, this, and not the previous alternative case, will be that of actual partially liquefied homogeneous matter, *i.e.* of liquefied crystals the ultimate parts of which retain some kind of stable structure. Cases in which the fragments are not all of the same pattern are probably furnished by crystals containing "water of crystallisation," and by double salts.

similar,[1] we easily see that although the situation of the boundaries of these fragments is, in the case of any given type of homogeneous structure, quite indeterminate so long as the precise nature of the structure is unknown, we are able to range the infinite number of ways in which the given type is divisible symmetrically into similar fragments in certain distinct classes; and this applies whichever of the two alternative kinds of disintegration above defined obtains.

For in the case of each type all the infinite number of possible ways in which such a subdivision can take place can be classed in groups *according to the coincidence-movements (Deckbewegungen) found surviving in each of the isolated similar fragments or parts of the disintegrated assemblage.* And taking into account the possession by some of the structures of the additional property of identity with their own mirror-images, we may further distinguish cases in which each fragment is identical with its own mirror-image from cases in which the fragments are enantiomorphs, and consequently half of them right handed and the other half left-handed.[2]

Thus, for example, if a certain type of homogeneous structure contains a single set[3] of digonal axes of rotation, and a single set of trigonal axes of rotation, we can discriminate cases in which each fragment has a digonal axis and no trigonal axis from those in which there is a trigonal and no digonal axis, and both these again from those in which the fragment when isolated has no axis at all.

As to cases where there is identity with the mirror-image. If a type of homogeneous structure contains either centres of symmetry (inversion centres), or planes of symmetry, on the two sides of which parts enantiomorphically related are *directly opposite*, or centres about which parts thus related are arranged as in types 63c and 64c in my list,[4] *we can*

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[1] That is, either identically or enantiomorphically similar. Comp. cases of racemic compounds referred to above, p. 129.

[2] Cases are conceivable in which the enantiomorphic property of fragments is manifested only when they are considered with respect to the unbroken structure; they may themselves, considered alone, be destitute of this property. In other words, groupings which are identical with their own mirror-image, and all identically alike, may occupy two different sets of enantiomorphically similar situations in a structure. Indeed, the groupings which survive in the liquid state may, when considered alone, apart from the structure, have *any additional elements of symmetry which are compatible with those which they already possess as parts of the structure.*

[3] Axes are of the same set when their relation to the entire system of axes found in the structure is the same, *and that whether they can be brought to coincidence or not.*

[4] See *Zeitschr. für Kryst.* XXIII. p. 59.

have either kind of fragment produced by disintegration, that which is identical with its own mirror-image, or that which occurs always in two sorts, a right handed and a left handed. If, however, neither of the elements of symmetry just referred to is present, but the enantiomorphic correspondence of parts is solely due to the existence of those planes of symmetry (Gleitebenen) on the two sides of which related portions are *not directly opposite, the existence of finite fragments which as they lie in the structure are identical with their own mirror-image is precluded.*[1]

For if a portion A on one side of such a plane were joined in the same fragment with an enantiomorphically similar portion A′ situated on the other side of the plane, the existence of the translation of A requisite to bring it opposite to A′ would necessitate the existence of a similar translation in the *same direction and similarly related to the structure* of A′, which would therefore bring the latter opposite to a portion identical with A in form but not in position, and this third portion *must therefore, for the sake of symmetry, also form part of the same fragment.*

Consequently, unless an infinite chain of enantiomorphically related portions A, A′, &c. is found in the same fragment, no two such can in the last mentioned case be united *without impairing the symmetry of the structure*;[2] and, failing union of this kind, no fragment identical with its own mirror-image can be formed.

It is possible to state the method by which the classification referred to is to be accomplished in what is perhaps a conciser and plainer way.

In another place I have defined singular point-systems as those whose points consist of fewer points, and these in more symmetrical situations than are occupied by the other point-systems of a homogeneous structure.[3] *The different types of partitioning can be distinguished according to the nature of the central or of some most symmetrically placed point in one of the similar fragments, each different kind of singular point found in any given type of homogeneous structure forming the central or most symmetrically placed point of a distinct type of fragment, and there being in addition to the types thus distinguishable one other type of fragment in each case, that which has no point in it which the nature of the type of structure requires shall be more symmetrically situated than the rest.*[4]

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[1] They may, however, considered apart from the structure, possess the property as a special peculiarity not derived from it. Comp. note 2, p. 131.

[2] Comp. *Zeitschr. für Kryst.* XXV. p. 88.

[3] *Zeitschr. für Kryst.* XXIII. p. 60.

[4] The least symmetrical type of all (hemipinakoidal-anorthic), as it possesses no singular points, presents but one type of partitioning for simple structures, *i.e.* for structures whose ultimate fragments are all similar.

The problem of finding the different types of partitioning possible resolves itself therefore into the problem of finding the different kinds of singular points present in each type of homogeneous structure.

The solution of the latter, as I have shown in the paper already referred to, depends on ascertaining precisely certain features of each type of structure, viz.:—

1. The different sets[1] of axes of rotation present.

2. The centres of symmetry.

3. The planes of symmetry which have the enantiomorphically related points directly opposite on their two sides.

4. The symmetrically placed planes with accompanying centres, such as those of the two types of structure identical with their own mirror-images, which have no centres of symmetry or planes of symmetry.[2]

5. The points and lines of intersection of these various elements of symmetry.

The table given below shows the result of applying the method of classification described to a few of the types of homogeneous structure belonging to the regular or cubic system; but before passing to this it may be helpful to state some conclusions as to the general features of the similar fragments produced by symmetrical partitioning.

The fact that no translation (Schiebung) or screw movement (Schraubung) can bring a fragment to coincidence with itself, shows that the only coincidence movements (Deckbewegungen) with which we have to deal are *rotations* of the unbroken structure.[3]

No fragment can, it is evident, have more than one axis of rotation *with the same direction;* and if it possesses more than one, inclined one to another, all such axes must intersect in a single point.

If a fragment possesses a digonal axis of rotation and no other, it will contain two identically similar points of every kind found in it, except those lying on the axis.

If a trigonal	axis and no other,	3,	except those lying on the axis.		
tetragonal	,,	,,	4,	,,	,,
hexagonal	,,	,,	6,	,,	,,

[1] See note 3, p. 131.

[2] There are some such planes in other types of structure besides the two 63c and 64c here referred to, which form a group by themselves. Comp. *Zeitschr.* XXIII. 59, XXV. p. 90.

[3] A fragment may, as we have intimated, have an axis or axes not found in the unbroken structure, but this is outside our province here, as we are not making a classification based on individual peculiarities of the fragments, but one based on the general features of the different types.

If two axes of rotation which are *not* both digonal axes intersect, it is evident that other similar axes must also pass through the point of intersection, the positions of which are discovered by carrying out movements about the two given ones; but the number of identical points involved does not equal the product of the co-efficients of all the intersecting axes. Thus the centre of a "24 punkter" of Sohncke is the point of intersection of six digonal, four trigonal and three tetragonal axes.

For each set[1] of axes of rotation in a homogeneous structure there will be at least one distinct corresponding group or type of varieties of partitioning, whose similar fragments possess such an axis. Also at least one distinct group for each different type of intersection of two or more axes, if there be any intersections, and at least one besides these of cases where the fragments possess no axes.

As regards those cases where the structure is identical with its own mirror-image, a plane of symmetry of a fragment, or any plane through its centre of symmetry, if it have one, will divide the fragment into two similar parts. If the fragment has an axis it also must evidently be central.

In no case can the fragment taken alone possess a plane of symmetry on the opposite sides of which corresponding points are not directly opposite (*i.e.* no Gleitebene), because such a property would, as already explained,[2] require the fragment to be infinite in one direction.

If an axis of a homogeneous structure lies in a plane of symmetry, the halves of the space units lying around the axis have contact with those of the same hand in this line only. It follows that when the fragments of the structure are such that this axis survives in them, they must, in order to be continuous *in substance*, also possess the plane of symmetry.

Summary.

The above investigation shows:—

1. The nature of homogeneity of structure, and the properties which distinguish it from structureless homogeneity. The new definition of a homogeneous structure recently put forward by the author in Groth's *Zeitschrift* is given.

2. A method of realising in a concrete form, and with great generality, the kind of repetition in space which constitutes homogeneity of

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[1] See Note 3, p. 131.

[2] See above, p. 132.

structure, the models employed for this purpose each consisting of a number of similar plaster hands appropriately arranged in space.

The total number of types of arrangement, all of which can be represented in this way, is 230, this being the number of typical point systems described by Fedorow and Schönflies, derived by their extension of Sohncke's methods. The various types of homogeneous structure, like the corresponding point-systems, all fall into the 32 classes of crystal symmetry.

3. What property common to all homogeneous structures whatever most nearly corresponds to Thomson and Tait's definition of homogeneity.

4. Reasons for regarding as untenable the arguments put forward by Fedorow in support of his recent attempt to select from among the types of homogeneous structure those which are possible for crystals, and to determine the shapes of their ultimate units.

5. The possibility of so classifying all conceivable ways of symmetrically partitioning all the types of homogeneous structure as to avoid all reference to the nature of the cell-faces, whether plane or otherwise, and, in other respects also, be perfectly general. Some reasons for undertaking this classification, notwithstanding its complexity, are given, the chief one being the relation of symmetrical partitioning to some stereo-chemical and other experimental facts.

FRAGMENT OF A TABLE OF THE VARIOUS KINDS OF SYMMETRICAL SINGLE PARTITIONING WHICH CAN BE MADE OF THE TYPES OF HOMOGENEOUS STRUCTURE.

Type partitioned.	No. of Variety.	Elements of Symmetry of the Homogeneous Structure which survive in a Cell or Element of the Partitioning.
1	(1)	A trigonal axis
,,	(2)	None
$1a_1$	(1)	A trigonal axis and a centre of symmetry
,,	(2)	A trigonal axis only (the fragments are enantiomorphs)
,,	(3)	None (fragments enantiomorphs)
2	(1)	A trigonal axis
,,	(2)	A digonal axis
,,	(3)	None
$2a_1$	(1)	A trigonal axis and a centre of symmetry
,,	(2)	A trigonal axis only (fragments enantiomorphs)
,,	(3)	A digonal axis ,, ,,
,,	(4)	None ,, ,,
$2b_1$	(1)	A trigonal axis only ,, ,,
,,	(2)	A digonal axis ,, ,,
,,	(3)	None ,, ,,
3 or 4	(1)	A trigonal axis and a digonal axis
,,	(2)	A trigonal axis only
,,	(3)	A digonal axis only
,,	(4)	None

FRAGMENT OF A TABLE OF THE VARIOUS KINDS OF SYMMETRICAL SINGLE PARTITIONING WHICH CAN BE MADE OF THE TYPES OF HOMOGENEOUS STRUCTURE—*Continued.*

Type partitioned.	No. of Variety.	Elements of Symmetry of the Homogeneous Structure which survive in a Cell or Element of the Partitioning.
5	(1)	A trigonal axis and a digonal axis
,,	(2)	A trigonal axis only
,,	(3)	A digonal axis only, of the kind which intersects a trigonal in the unbroken structure
,,	(4)	A digonal axis of the other kind
,,	(5)	None
$5a_1$	(1)	A trigonal axis and a centre of symmetry
,,	(2)	A trigonal axis and a digonal axis (fragments enantiomorphs)
,	(3)	A trigonal axis only (fragments enantiomorphs)
	(4)	A digonal axis only, of the kind which intersects a trigonal in the unbroken structure (fragments enantiomorphs)
,,	(5)	A digonal axis of the other kind (fragments enantiomorphs)
,,	(6)	None (fragments enantiomorphs)
6	(1)	Four trigonal and three digonal axes
,,	(2)	A trigonal axis only
,,	(3)	A digonal axis only
,,	(4)	None
$6a_1$	(1)	Four trigonal axes, three digonal axes, and a centre of symmetry, with accompanying three planes of symmetry
,,	(2)	A trigonal axis only (fragments enantiomorphs)
,,	(3)	A digonal axis and a plane of symmetry
,,	(4)	None (fragments enantiomorphs)
$6a_2$	(1)	Four trigonal and three digonal axes (fragments enantiomorphs)
,,	(2)	A trigonal axis and a centre of symmetry
,,	(3)	A trigonal axis only (fragments enantiomorphs)
,,	(4)	A digonal axis only ,, ,,
,,	(5)	None ,, ,,
$6b_1$	(1)	Four trigonal axes, three digonal axes, and six similar planes of symmetry
,,	(2)	A trigonal axis and a plane of symmetry
,,	(3)	A digonal axis only (fragments enantiomorphs)
,,	(4)	None ,, ,,
$6b_2$	(1)	Four trigonal and three digonal axes (fragments enantiomorphs)
,,	(2)	A trigonal axis only (fragments enantiomorphs
,,	(3)	A digonal axis only ,, ,,
,,	(4)	None ,, ,,
		&c. &c.