Addition to a former note[1] on the rotation of points and planes about an axis.

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THE evaluation of the determinant

$$\Delta_1 = \begin{vmatrix} l_1 & m_1 & n_1 \\ \cos\lambda & \cos\mu & \cos\nu \\ \frac{x}{OP} & \frac{y}{OP} & \frac{z}{OP} \end{vmatrix}$$

occurring at the foot of page 345, Vol. XII, can be effected without first squaring it. Multiply Δ_1 by

$$(1+2\cos\alpha\cos\beta\cos\gamma-\cos^2\alpha-\cos^2\beta-\cos^2\gamma)^{\frac{1}{2}},$$

say $D^{\frac{1}{2}}$, which is the value of the determinant

$$\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}.$$

The result is

$$\Delta_1 D^{\frac{1}{2}} = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\omega_1 & \cos\omega_2 & \cos\omega_3 \\ \cos\theta_1 & \cos\theta_2 & \cos\theta_3 \end{vmatrix}.$$

Similarly Δ_2 and Δ_3 may be evaluated.

Further D_1 (p. 347) is the value of $p^{-1}\Delta_1$ after ph/a, pk/b, pl/c have been substituted for $\cos\theta_1$, $\cos\theta_2$, $\cos\theta_3$ respectively. Accordingly[2]

$$D_1 D^{\frac{1}{2}} = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\omega_1 & \cos\omega_2 & \cos\omega_3 \\ h/a & k/b & l/c \end{vmatrix}.$$

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[1] Min. Mag. 1900, Vol. XII, p. 343.

[2] This result is due to Mr. T. J. I'A. Bromwich, M.A., Fellow of St. John's College, Cambridge.

Thus the final formulæ (7) are :

$$\frac{h'}{a} - \frac{h}{a}\cos\phi =$$

$$\sin\phi\, D^{-\frac{1}{2}} \begin{vmatrix} 1 & \cos\omega_1 & h/a \\ \cos\gamma & \cos\omega_2 & k/b \\ \cos\beta & \cos\omega_3 & l/c \end{vmatrix} - \cos\omega_1 (1-\cos\phi)\, D^{-1} \begin{vmatrix} 1 & \cos\gamma & \cos\beta & h/a \\ \cos\gamma & 1 & \cos\alpha & k/b \\ \cos\beta & \cos\alpha & 1 & l/c \\ \cos\omega_1 & \cos\omega_2 & \cos\omega_3 & 0 \end{vmatrix};$$

with similar expressions for k'/b and l'/c.

The formulæ of transformation for the case in which the angle of rotation is 180° can be found as follows :—

Since the point N is the middle point of PP', its projection on each axis is midway between the projections of P and P' on the same axis :—

$$\therefore\ OP\cos\theta_i + OP'\cos\theta_i' = 2\, ON\cos\omega_i;$$

$$\therefore\ \cos\theta_i + \cos\theta_i' = 2\cos\omega_i\cos PON.$$

Let as before the plane

$$hx/a + ky/b + lz/c = 1$$

$$\text{become } h'x/a + k'y/b + l'z/c = 1.$$

Then we must put $\cos\theta_1 = \rho h/a$, &c. ; $\cos\theta_1' = \rho h'/a$, &c.

Hence $$\frac{h+h'}{a\cos\omega_1} = \frac{k+k'}{b\cos\omega_2} = \frac{l+l'}{c\cos\omega_3} = 2\frac{\cos PON}{\rho};$$

where $\frac{\cos PON}{\rho}$ is given by equation (8) of page 347, Vol. XII.