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An improved form of Three-Circle Goniometer.

(*With Plate I.*)

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IN a previous paper[1] the author pointed out the advantages of a goniometer possessing three graduated circles, and described an instrument of this type which had resulted from the addition of a two-circle arrangement to an ordinary reflective goniometer with a single (horizontal) circle. As was remarked in that paper, the adapted instrument has unavoidably the disadvantage that measurements can be made through little more than a right angle in any particular zone, with the single exception of the zone of reference, without readjustment of the vertical and third circles. The zone may, indeed, be completed by measuring from the diametrically opposite pole in the zone of reference; but difficulties occasionally arise in practice, and it is convenient to be able to measure through at least two right angles round the free end of the crystal by rotation of the horizontal circle only. In a subsequent note[2] the author suggested a way of overcoming this difficulty; and in fulfilment of the hope therein expressed he describes in the present paper an instrument, in the construction of which the

[1] 'A three-circle goniometer,' Min. Mag., 1899, vol. xii, pp. 175–182; a translation appeared in Zeits. Kryst. Min., 1900, vol. xxxii, pp. 209–216.

[2] 'Preliminary note on an improved form of three-circle goniometer,' Min. Mag., 1901, vol. xiii, pp. 75–76.

suggested improvements with some modifications have been incorporated. The instrument has been constructed for the British Museum by Messrs. Troughton & Simms, of London.

As regards the arrangement of the graduated circles there is in principle no change at all, and the alterations relate solely to the optical parts of the instrument. The *line of reference*, i.e., the line to which the face under observation is brought perpendicular, is reflected so as to be nearly at right angles to the ordinary position. Thus on rotation round the axis of the horizontal circle the axis of the vertical circle may coincide with the line of reference in two positions, and observations may be obtained of both the faces of the crystal which are at right angles to this axis; previously no observation was possible from the face next to the vertical circle.

In order to effect this change, an auto-collimating telescope was at first employed, and the direction of light bent by means of a reflecting prism placed before the object-glass. This arrangement, however, was found in practice to introduce several difficulties: the stray light reflected from the back of the object-glass caused a glare in the field of view which completely prevented observation of minute crystalline faces; further, accompanying the principal image of the object-slit were to be seen a series of 'ghosts,' i.e., the images due to multiple reflections at the crystal-face and the near face of the prism; lastly, the field of view of the telescope was not as large as is desirable. These difficulties were, indeed, to some extent overcome; but after some trial it was finally decided to use separate tubes for the telescope and collimator, lying alongside each other, and to replace the single glass-prism by two distinct mirrors, one for each tube. The use of separate reflectors secured greater facility of adjustment.

(*a*) *The principle of a goniometer with three graduated circles.*

The object of a three-circle goniometer is to bring any desired edge on the crystal without readjustment parallel to some arbitrary direction in space, and it may be effected in various ways: but undoubtedly in practice the most convenient method is to maintain the optical parts, and therefore the eye of the observer, in the same position and to assign all the motions to the part carrying the crystal. The crystals usually selected for measurement are small, and, their weight being inconsiderable, they are held rigidly by the wax during the course of the observations. In the case of a goniometer constructed for the measurement of large crystals and especially of crystals still in position on the matrix, it would be advisable to give two of the motions to the optical parts. In such an instrument the specimen would be rotated about an horizontal axis only, and the bearing cones of the axis would be relieved of unnecessary strain by means of a counterbalancing spring placed underneath the stage.

In the present instrument all the motions have been given to the crystal, and the goniometer possesses essentially three graduated circles arranged thus:—

A the horizontal circle, whose axis is vertical and fixed in space;

B the vertical circle, whose axis is movable in an horizontal plane; and

C the third circle, whose axis may assume any desired direction in space.

These circles must satisfy the following conditions:—

The axis of *A* must be at right angles to the line of reference;

The axis of *B* must be at right angles to that of *A*;

The axis of *C* must be at right angles to that of *B*, although inclinable at any angle to that of *A*; and finally

All three axes must pass through the same point, the *optical centre*, which must itself lie on the line of reference.

How these conditions may be fulfilled will be considered after the description of the instrument.

(*b*) *The construction of the instrument.*

The base of the goniometer is formed of a tripod resting on three levelling screws. Rigidly connected with it are the arms carrying the pair of microscopes used for reading the scale of the circle *A* and the pillar supporting the tubes of the telescope and collimator. The latter tubes lie within outer tubes, each of which is adjustable about a vertical axis and about an horizontal axis at right angles to the length of the corresponding tube by means of three screws, one pushing and two pulling, which operate on the fixed collar. The tubes themselves may revolve about, and be translated in the direction of, their length; they are held by means of tangent screws which grip the split ends of the outer tubes. A small reflecting prism is placed immediately outside the slit of the collimator to permit of illumination from the side.

At the ends of the telescope and collimator remote from the observer are two tubes carrying mirrors, which fit round, and may slide along, the optical tubes; the ends away from the mirrors are split and the two halves are fitted with tangent screws to permit of rigid clamping to the corresponding optical tubes. Each mirror has three adjustments: a revolution about the length of the tube containing it, a translation in this direction, and finally a small adjustment about a vertical hinge; the last movement being actuated by means of a screw working against a spring. The telescope-mirror is placed further away than the collimator-mirror, and its tube is cut away at the sides so as to permit the passage of light reflected from the latter mirror. The mirrors are composed of speculum-metal, which gives reflections comparable in brightness with those obtained from totally-reflecting glass-prisms and avoids the difficulty due to stray reflections or 'ghosts.' If the metal becomes tarnished, it is easily cleaned by means of cotton-wool dipped in a mixture of two parts of alcohol and one part of ammonia; methylated spirit is not suitable because of the film frequently left on evaporation.

The Signal.

On the top of the tube containing the telescope-mirror is a recess, which receives an achromatic lens when not in use. The lens is hinged about an horizontal axis, and a lever without the tube enables the observer to place it in front of the object-glass of the telescope, thus converting the latter into a microscope of low power for viewing the crystal under measurement. Another lens is carried on an arm, which is hinged on a slide movable along a bar placed on the top of the telescope-tube near the eyepiece, and may be placed in front of the eyepiece; in this way smaller magnification is obtained, and the crystal is visible when not accurately placed at the optical centre.

The cross-wires in the telescope are fixed in position. Three eyepieces of varying powers are provided for viewing the reflected image. The type of signal employed (see fig.) differs slightly from that in ordinary use, because the image has to be accurately placed with respect to the horizontal as well as with respect to the vertical wire. The width may be altered at will; but once set it should not be altered, since the zero-reading of the circle A is affected at the same time.

The circle A has a diameter of 18 cm., approximately, and rotates about a vertical axis. Underneath its stage is a countervailing spring, by means of which the bearing surfaces of the cones may be relieved of as much of the superincumbent weight as may be desired. Inside the plate, on the bevelled edge of which the circle A is graduated, is another cone, larger in diameter but shorter in length, which is rigidly connected with the thick plate which carries the remaining circles and their accessories. The latter portion of the instrument may thus be rotated independently of *A* and placed in any position at will with respect to it; a strong clamp is provided for holding the two plates firmly together when in their proper relative position. By means of this device, as will be explained below (p. 11), the zero-readings of *A* may be made very nearly 360° and 180°. The scale is divided to 10′ and is read by means of a pair of microscopes placed diametrically opposite each other. In the focal plane of each is a pair of spider-lines, whose distance apart is equal to the apparent width of the divisions on the scale; they are operated by a micrometer-screw. One revolution of the screw carries the spider-lines from one to the next division on the scale, and thus corresponds to an angle of 10′. Its head is divided into ten parts, each of which is subdivided into six intervals, which therefore correspond to tens of seconds; hence by estimation the circle may be read to single seconds of arc.

The arm carrying the circle *B* is firmly attached to the plate mentioned above, by means of four screws, on releasing which the whole piece may be adjusted by means of other screws either towards the axis of *A* or at right angles to this direction. The fixed limb, carrying a pair of microscopes for reading the scale of *B*, is attached to the arm by means of three adjusting screws. Each pair consists of a larger screw of low pitch operating on the arm, and of a smaller screw, passing through the former and holding the microscope-limb firmly against the blunt points of the larger screw. The adjustment is made by moving the larger screws through the desired amount; the smaller screws are then tightened. By this device the pull on the limb is direct and there is no tendency to twist. Both sets of screws have hexagonal heads and are turned by means of the spanners shown at the base of the instrument (Plate I). The scale is 11 cm., approximately, in diameter, and is read in the same way as that of *A* by means of a pair of microscopes to 10″ directly, and to single seconds of arc by estimation.

To prevent excessive wear of the lower half of the fixed cone of the circle *B* owing to the weight bearing on it, two pairs of friction wheels are provided: one pair lies on the fixed limb between it and the circle, and is operated by means of a single balanced spring, to assure the pressure being always vertical; the other pair, visible in the picture, lies near the counterpoise and is operated by an adjustable spring. By means of pins passing through the fixed limb the former pair may be held out of the way, whilst the circle *B* is being placed in position, after removal for the cleaning of the bearings or some other purpose. The pin, on which the spring of this pair is pivoted, projects through the fixed limb, and may be raised, if it is found desirable to increase the countervailing pressure on the movable cone.

The arm, carrying the circle *C*, and the inside cone of the circle *B* are formed of a single casting to ensure the greatest possible rigidity. The latter circle is firmly attached to the arm, and the relative positions of the circle and the arm cannot, therefore, be altered, as is the case with the horizontal circle and the corresponding plate, for the purpose of making the zero-reading of the scale of *B* some convenient quantity. It has, however, been arranged that this reading should be approximately 180°, or 360° (depending on the microscope used), and it can be made almost exactly so by moving the microscopes; for this purpose the holes, through which pass the screws holding the microscopes to the fixed limb of the circle *B*, have been slotted.

The circle *C* is read to minutes of arc by means of a pair of oppositely placed verniers; greater refinement is unnecessary because more exact measurements may be made by means of the circle *A*. The fixed limb, carrying the verniers, is attached to the arm by means of three pairs of adjusting screws as before. Its scale has a diameter of 9 cm., approximately, and the cones taper towards the crystal-holder, since this is the usual tendency of the weight.

All three circles have the ordinary type of clamp and slow motion. The crystal-holder too is of the usual form, and possesses all the adjustments required. The rods, on which the crystals to be measured are fixed, are grooved down their length, so that any rod may be removed from the goniometer and set up again in very nearly the same position.

Counterpoises are provided to maintain the pressures evenly on the bearing cones of the circles *A* and *B*; that balancing the third circle *C* and the crystal-holder has been placed on the other side of the circle *B*, any shear on the cones of the circle *B* being thus obviated.

The mirrored axes of the telescope and collimator are inclined to each other at an angle of 22°, approximately. The greatest possible rotation about the axis of *A* is 240°, approximately; the axis of *B* can make with the line of reference angles of 57° towards the telescope and 3° towards the collimator respectively. When *B* is next to the collimator and its axis coincides with the line of reference, the greatest rotation about its axis is 188°, approximately, the axis of *C* making with the vertical, angles of 53° and 135° towards the collimator and the circle *A* respectively; when *B* is on the other side, the greatest rotation is 200°, approximately, the axis of *C* making the same angle towards the circle

A, but an angle of 65° towards the telescope. The last angle might with advantage be made at least 90°, which, indeed, was the original arrangement before the alteration of the optical parts, by placing the tubes of the telescope and collimator closer together.

The tripod, main stage, and the circles are constructed of gun-metal; for the arm carrying the circle *C* and for the microscope-arms phosphor-bronze has been used; and the central rod of the crystal-holder is made of steel.

(*c*) *The adjustments.*

We may now consider the method of effecting the necessary adjustments.

To adjust the line of reference perpendicular to the axis of *A*, it is convenient, though in this type alone of goniometers not necessary, to use a plate of glass, the sides of which are accurately parallel. Such plates are supplied with goniometers by the makers and are as a rule correct enough for the purpose; nevertheless this has always been the weak point of the adjustments of such instruments, because the parallelism of the plate has to be assumed and is beyond the control of the observer.

Although not essential, it is convenient to have the mirrored axes of the telescope and collimator, and not merely the line of reference which bisects their acute angle, at right angles to the axis of *A*. The adjustment is effected in the usual way. The parallel-sided glass-plate is mounted on the crystal-holder, and adjusted as accurately as the eye can judge so that its plane is parallel to the axis of *A*. Care must be taken that the circle *B* is in such a position that a semi-revolution about the axis of *A* is possible. A glass-plate, the sides of which need not be parallel or even plane—a microscope-slide serves the purpose well enough—since no serious error is thereby introduced, is held in such a position in front of the eyepiece that light from the source of illumination is reflected down the tube of the telescope; the object being to illuminate the cross-wires and thus to see their images derived by reflection from the glass-plate. The images are not at first easily seen because of the confusion caused by the brilliant images of the source of illumination obtained by reflection from the surfaces of the various lenses in the telescope; and it is, therefore, convenient to use for the first rough adjustment the image of this source obtained by reflection from the plate; it is easily distinguished from the other images by being fainter and by moving with the plate. This image serves as a useful indicator for the images of the cross-wires, which may be seen after a little manipulation of the plate reflecting light down the tube. It may be remarked that, unless the cross-wires are set accurately in the focal plane of the telescope, their image thus derived will not come to focus in the same plane, and may, indeed, not be visible at all; this affords a test of the accuracy of the focusing of the telescope. The tube of the telescope-mirror is adjusted until the cross-wires and their images coincide. Rotating the plate through a semi-revolution about the axis of A, we view the images obtained by reflection from the other side of the plate: whatever be the interval separating the horizontal wire from its image, the plate must be adjusted through one half and the telescope-mirror through the other half. Turning back to the other side of the glass-plate, we repeat the adjustment. The agreement is soon made satisfactory.

The next step is to adjust the mirror of the collimator until the image of the signal, when in the centre of the field, is bisected by the horizontal wire. It may happen that on rotation about the axis of A the centre of the image does not traverse the horizontal wire, or in other words the mirrored wire is not perpendicular to this axis: we must rotate the tube of the telescope and at the same time the mirror-tube through the same angle in the opposite sense until the adjustment is effected. A similar adjustment must be made in the case of the collimator, should the image not be upright.

To adjust the axis of B perpendicular to that of A, we retain the glass-plate in the same position as before, and rotate it about the axis of C, until it is roughly at right angles to that of B: then we clamp C. By means of the crystal-adjustments and the slow-motion of C we set the plate in such a position that on rotation about the axis of B there is no movement of the image of the signal. For this adjustment we require at least a semi-revolution about the axis of B. We now adjust the axis of B with respect to that of A by means of the screws provided for the purpose until the image is exactly bisected by the horizontal wire.

The axis of C may be adjusted perpendicular to that of B in precisely the same way: the axis of B is rotated about that of A until it makes a right angle with its former position, and a side of the plate is brought exactly perpendicular to the axis of C, so that there is no movement of the image obtained by reflection from it. The axis of C is now adjusted until the image is bisected by the vertical wire; if at the same time it is bisected by the horizontal wire, the reading of B gives us the horizontal position of the axis of C. An equally effective and, since we may retain the glass-plate in its initial position, more convenient method is to make the axis of *C* coincide with that of *A*. Then since the latter is perpendicular to that of *B*, so must the former be. Keeping the axis of *B* in coincidence with the line of reference, we rotate the glass-plate about the axis of *C*. The images from the two sides will probably not both be bisected by the horizontal wire. By means of the crystal-adjustment make one lie as far above as the other is below; then by means of the adjustment of *C* adjust the axis until one image, and concurrently the other, is symmetrically set with respect to the horizontal wire. Rotating round the axis of *A* through a right angle and finding the reading of *B* when the image is set on the same wire, we obtain the zero-reading of *B*, i. e., the reading corresponding to the vertical position of the axis of *C*. The great advantage of this method is that it enables the observer to check any adjustment of direction without altering the position of the glass-plate.

In the final adjustment, the parallelism of the glass-plate may be disregarded. As has been pointed out earlier in this paper (p. 2), the axis of *B* may be parallel to the line of reference, or, if correct in absolute position, may coincide with it, in two positions, the circle *B* lying first on one, and then on the other, side of the optical parts. In either position we may adjust accurately some plane surface, say a side of the glass-plate hitherto employed, perpendicular to the axis of this circle. It is clear then that, if the images in the two cases have been symmetrically set on the cross-wires, the corresponding planes have occupied exactly parallel positions. We now adjust the axis of *B* with respect to that of *A* until the two images are bisected by the horizontal wire.

To adjust the axis of *C*, we turn from one position to the other through a semi-revolution about the axis of *A*, and then rotate *C* also through a semi-revolution, so that the image obtained by reflection from the same side of the glass-plate as before is on the cross-wires: whatever be the interval separating the centre of the image from the horizontal wire, we adjust the plate through one half and the axis of *C* with respect to that of *B* through the other half. The accuracy of the adjustment is tested again in the initial position. Turning round the axis of *A* until that of *B* is at right angles to the line of reference, we rotate the glass-plate about the axes of *B* and *C* until the image is set symmetrically on the cross-wires. The axes of *A* and *C*, if correct in absolute position, now coincide; and if, clamping *B* in this position, we rotate about these axes so as to retain the image in the field of view, it should always be bisected by the horizontal wire.

The axes of the three circles are now accurately placed as regards direction; and it only remains to adjust them in absolute position so that they pass through the same point, the optical centre, which must itself lie on the line of reference.

The glass-plate is replaced by a rod, sharpened to a point at the free end, and the telescope is converted into a microscope by means of the auxiliary lens which is placed before the object-glass. The point is raised or lowered until it is somewhere near the centre of the field, and adjusted until there is no shift on rotation about the axis of *A*. Should the point not lie on the vertical wire, the telescope must be adjusted accordingly by translation in the direction of its length; the final adjustment should be made by turning the mirror about its vertical hinge. Since it is impossible to make this translation without affecting the position of the line of reference, the needle-point should be brought beforehand just on to the horizontal wire, and after adjustment has been made, the point should be again set on the wire; with this precaution the readjustment of the line of reference required will be small.

We next raise or lower the needle-point until there is no shift on rotation about the axis of *B*. If the point now lies off the horizontal wire, we turn the collar containing the telescope-tube about its horizontal axis accordingly. If, when the axis of *B* is parallel to the line of reference, the point lies off the vertical wire, we release the screws holding the base of the arm carrying the circle *B*, and move its axis parallel to itself horizontally by means of the screws provided until the point is exactly on the vertical wire after the clamping screws of the base have been securely tightened; this adjustment may, if preferred, be effected by means of the two double-screws lying in an horizontal plane, which hold the fixed limb carrying the pair of microscopes firmly to the arm just mentioned.

The final step is to adjust the needle-point until there is no shift on rotation about the axis of *C*. If, on turning round the axis of *A* until that of *B* is at right angles to the mirrored axis of the telescope, we find that the point lies off the vertical wire, we again release the screws holding the base, and move the arm to or from the axis of *A* as may be required. If, again, when the axis of *B* is parallel to the mirrored axis of the telescope, the point lies off the vertical wire, we adjust the axis of *C* accordingly so that it moves towards that of *B*.

We have only to adjust the collimator in absolute position so that its mirrored axis passes through the optical centre. To do this, we adjust the glass-plate so that the image obtained by reflection from it lies symmetrically on the cross-wires, and, removing the eye-piece, we glance down the tube of the telescope. A portion of a circular patch of light is seen, due to the reflection of the illuminated object-glass of the collimator. If we adjust the collimator until this patch lies symmetrically with regard to the object-glass of the telescope, we shall effect the adjustment with sufficient accuracy for the purpose.

We must make a final test of all the adjustments, and the goniometer is ready for use.

For convenience of discussion, the various adjustments have been taken in the above order; but in actual practice we should make the absolute adjustment immediately after the corresponding relative adjustment, and thus, as far as possible, avoid interference with adjustments already made.

(*d*) *The method of using the instrument.*

We must first determine the exact zero-readings of the circles *A* and *B*. It was pointed out above (p. 5) that the upper part of the instrument could be rotated about an interior cone independently of the circle *A*. Rotating, then, *A* until the microscope next the observer reads exactly 360°, we turn the upper part until the axis of *B* coincides with the line of reference and the circle *B* lies on the side of the crystal-holder remote from the optical parts and clamp the parts together. Whatever correction may now be found necessary will only be a few seconds. Next we find the position in which the axis of *C* is vertical, and move the microscopes provided for reading the scale of *B* until they read as nearly as possible 360° and 180° respectively. A small correction may be found necessary, but it will only amount to a few seconds.

To make the best use of the goniometer, we adjust the edge of some prominent zone, preferably one of symmetry, parallel to the axis of *C*. To effect this, the axis of *B* is brought into coincidence with the line of reference; at the same time the axis of *C* should be vertical, since in that position the images obtained by reflection from the faces of the zone, when in adjustment, will cross the field horizontally on rotation of this axis, and the adjustment is made with greater facility. The zone thus adjusted is termed the *zone of reference*. We rotate *C* and take the readings of this circle for every face in the zone of reference, the axis of *B* still coinciding with the line of reference. Each reading corresponds to the position in which the particular face is perpendicular to the axis of *B*, and hence on rotation about this axis that face always remains parallel to the same plane. The faces in the zone of reference may conveniently be termed *origins*. The readings of the circles *A* and *B* for any other face of the crystal give us, after any correction required has been made, the distance and azimuth measured from the origin and the zone of reference, respectively. If the zone under measurement does not intersect the zone of reference in an existing face, the reading of *C*, taken in conjunction with the readings already found for the origins which have been encountered, will give the position of the pole of intersection in the zone of reference. The circle *C* can be read to minutes only; more exact measurements may be made on the circle *A*, if the axis of *C* be maintained in the vertical position.

Having measured the zone of reference, we study the observations and note the symmetry possessed by the zone. From a face of symmetry, if there be one, as origin, we systematically measure the crystal, commencing with the zone of reference and gradually increasing the azimuthal angle. A few measurements will soon show what symmetry the crystal apparently possesses. If the zone initially selected for reference is not a principal zone on the crystal, it is best to readjust the crystal so as to make such a zone that of reference.

Having completed the measurement of the crystal, we plot the positions of the poles of the observed faces on a sheet of paper. Whether the stereographic or gnomonic projection be selected, the plane of the diagram should be at right angles to the edge of the zone of reference. In general work the former projection is the more suitable. The author has found the stereographic net [1] devised by Professor E. von Fedorow for petrological determination, to be of great service and to give satisfactory results. The net is placed on a sheet of paper and the positions of the various poles marked by pricking through the net. Revolving the net about its central point, we may place the zero-point on any other origin, and read off from the net the distance and azimuth of any face referred to this new origin. If the zonal relations are very complicated, the gnomonic projection may with advantage be employed. By means of the table [2] prepared by the author, the positions of the various poles may be readily determined on any scale that may be suitable to the purpose. The net[3] drawn by Mr. H. Hilton may be used in a similar way.

[1] 'Über die von Professor E. von Fedorow herausgegebenen stereographischen Netze,' Leipzig, Wilhelm Engelmann, 1897. Vide Zeits. Kryst. Min., 1896, vol. xxvi, p. 337.

[2] Min. Mag., 1903, vol. xiii, facing p. 320; and Zeits. Kryst. Min., 1904, vol. xxxix, facing p. 154.

[3] Min. Mag., 1904, vol. xiv, p. 18.

The author has occasionally found it necessary to ascertain the co-ordinates of some face, when referred to an origin other than that from which it has been measured. In case it should not be worth while to remount the crystal, a glass mirror has been provided, circular in shape and swivelled about a diameter. The mirror, when used, is placed so that this diameter is as nearly as possible parallel to the axis of one of the cylindrical adjustments of the crystal-holder. The circles A and B are set to the known distance and azimuth (subject to any necessary zero-correction), and the mirror is turned about its own diameter and about the axis of C until the image, obtained by reflection from it, is symmetrically set on the cross-wires, the final adjustments being made by means of the crystal-adjustment and the slow-motion in the two cases. Rotating about the axis of C through the angular amount necessary to bring the new origin into the zero-position, we revolve the mirror about the axes of A and B until the image, obtained by reflection from it, is again symmetrically placed on the cross-wires, and from the readings of these circles determine directly the new distance and azimuth.

The above procedure is, of course, merely the solution of a spherical triangle, in which the two sides and the included angle are known. By means of the mirror it is possible to solve all cases with two exceptions; when the three sides or the three angles are known, a second mirror [1], inclinable at any desired angle to the first, is required.

The method of measuring crystals on this form of goniometer may be varied to suit the different cases that may arise. For example, in the case of an hexagonal crystal, in which the prism zone is well developed, we should select this zone for reference; but it would be simpler to take measurements from the basal face. To do this, we turn the axis of C round that of B until the former is at right angles to the axis of A. Then, clamping B and using the circles A and C, we derive from A the complements of the distances from the base and from C the azimuthal angles.

The measurement of twinned crystals is facilitated by the use of this form of goniometer; especially in those numerous cases, in which some one corresponding zone on the two individuals [2] is readily recognizable.

[1] Cf. E. von Fedorow, 'Universalgoniometer mit mehr als zwei Drehaxen und genaue graphische Rechnung,' Zeits. Kryst. Min., 1900, vol. xxxii, p. 481; and K. Stöckl, 'Das Fedorow'sche Universalgoniometer in der Construction von Fuess und die Anwendung dieses Instrumentes zur mechanischen Auflösung sphärischer Dreiecke,' Zeits. Kryst. Min., 1904, vol. xxxix, p. 42.

[2] Cf. Min. Mag., 1902, vol. xiii, p. 146; and Zeits. Kryst. Min. 1903, vol. xxxvii, p. 231.

In such a case, we set up one individual in the usual way and measure it for the identification of the faces, employing for reference the zone which has been recognized; we then turn to the pole of intersection with the corresponding zone on the other individual, and take measurements of both individuals from this pole as origin. The corresponding faces on the two individuals are equally distant from this origin, and are, therefore, easily recognized. The pole of twinning (corresponding to either a face or a zone-axis) is distant a right angle from this origin and its azimuth is half either the obtuse or the acute angle between the zones. Probably only one of these positions is near a crystallographically possible pole, but any doubt is at once removed by a study of a projection of the poles.

Readings of the origin should be taken for each zone in case it has shifted at all from the zero-position. Should it not be correctly adjusted, its pole on the sphere of projection will describe a small circle about the pole of the axis of *B*, and the greatest error introduced in the value obtained for the distance of any other pole from it by neglecting the shift is the distance between the two poles mentioned. In the calculation of the crystallographical constants, it may be best to utilize measurements of distances only, because in this way we measure directly from one image to another and the chance of including errors of adjustment is reduced to a minimum.

The makers of the instrument, Messrs. Troughton & Simms, of London, have in its construction fully maintained the high standard of mechanical skill characteristic of their work, and the author is especially grateful to them for the interest they have taken in the development of the goniometer, and for their readiness to incur any trouble in overcoming the various difficulties that are inevitably encountered in the realization of a new form of instrument.

As some indication of the accuracy that may be expected of this goniometer, the solutions of two pairs of spherical triangles, in which the two sides and the included angle are known, are appended, and the results obtained by observation with the instrument in the manner described in the paper (p. 13) are compared with the computed values. The known sides of the triangles in each pair are identically the same, but the included angles are supplementary. The data have been selected at random, and are given to minutes only, because the scale of the third circle does not admit of greater refinement. Measurements are made in turn from both of the poles, in which the given sides meet the third (unknown) side, and thus two independent values are obtained for this side; any serious discrepancy between the two would point to some error in setting or in observation.

Given $\begin{cases} b & 74^\circ 27' \\ c & 53\ 42 \\ A & 42\ 25 \end{cases}$

	Observed.	Calculated.	O.–C.
a	42° 57′ 4″	42° 57′ 11″	−7″
B	107 29 55	107 30 34	−39
a	42 56 50	42 57 11	−21
C	52 55 0	52 55 10	−10

$\begin{cases} b & 74^\circ 27' \\ c & 53\ 42 \\ A & 137\ 35 \end{cases}$

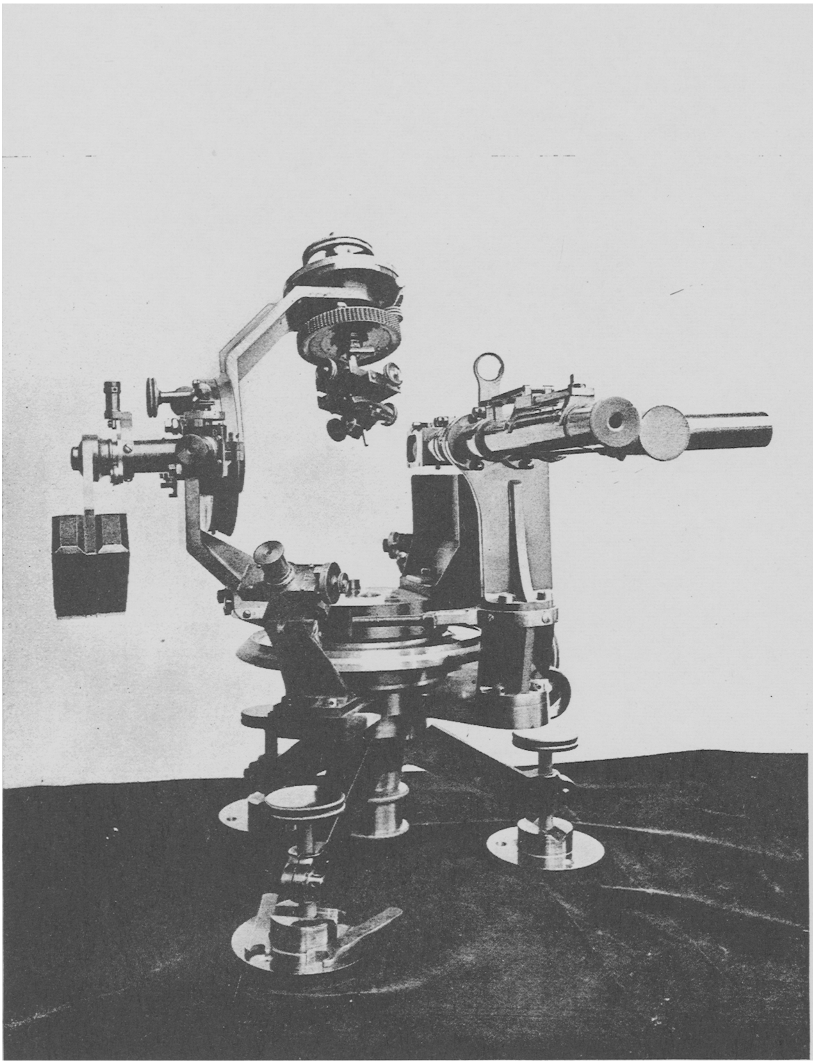
	Observed.	Calculated.	O.–C.
a	114° 29′ 40″	114° 29′ 16″	24″
B	45 33 57	45 33 58	−1
a	114 29 25	114 29 16	9
C	36 40 35	36 40 48	−13

Given $\begin{cases} b & 65^\circ\ 7' \\ c & 35\ 25 \\ A & 55\ 11 \end{cases}$

	Observed.	Calculated.	O.–C.
a	49° 58′ 22″	49° 58′ 44″	−22″
B	103 27 4	103 27 41	−37
a	49 58 0	49 58 44	−44
C	38 24 43	38 24 33	10

$\begin{cases} b & 65^\circ\ 7' \\ c & 35\ 25 \\ A & 124\ 49 \end{cases}$

	Observed.	Calculated.	O.–C.
a	87° 33′ 22″	87° 32′ 59″	37″
B	48 12 30	48 11 52	38
a	87 32 50	87 32 59	−9
C	28 26 17	28 26 17	0



G. F. HERBERT SMITH: THREE-CIRCLE GONIOMETER.