The construction and use of the Moriogram.

(*With Plate III.*)

By G. F. Herbert Smith, M.A., F.G.S.

Assistant in the Mineral Department of the British Museum.

[Read March 22, 1904.]

THE diagram (Plate III), to which for convenience the author has given the name moriogram[1], is intended to provide a method for rapidly determining the angles between a face of symmetry and all other faces with rational indices, which lie with it in some particular zone, when two of the angles are known, or, what is precisely the same thing, between a zone of symmetry and all other zones with rational indices, which intersect it in the same pair of opposite poles, when two of the angles are known: whenever possible it should be arranged that one of the known angles is a right angle. The moriogram may, therefore, be used in the course of computation for checking the accuracy of the work, or during the goniometrical measurement of a crystal for finding the relations subsisting between the faces or the zones.

The discussion—without, however, the loss of any generality—will be simplified if we consider a particular case, say that of the zone [*ab*], [100,010]. We will first suppose the angle *ab* (100 : 010) to be a right angle.

Let the angle *am* (100 : 110) be θ, then the angle between *a* (100) and any other face in the zone is $\tan^{-1} x$, where x is one of the values given by

$$x = y \tan \theta,$$

y being a simple whole number or a fraction composed of simple whole numbers.

If now we take x and y as Cartesian coordinates and assign to θ the values 0°, 1°, 2°, ... and so on, up to 90°, we obtain a pencil of rays emanating from the origin, each of which is inclined to its immediate neighbours at an angle of 1°.

We notice that, when $x = y$, $\tan \theta = 1$, and that therefore the unit distances on the two axes are equal. If the unit distance from the origin on the axis of Y be divided into rational parts, and lines be drawn through these points parallel to the axis of X, their intersections with the particular ray of the pencil in question give us at once the

[1] μόριον, a part.

angles required. If the unit distance on the axis of X be divided so that the distance of any point from the origin represents the tangent of the corresponding angle, we may read off from the diagram the angles mentioned above. Lines drawn through the point on this axis representing any particular angle intersect the corresponding ray of the pencil in the line $y = 1$.

As yet we have merely considered fractions less than unity, that is to say the indices of faces lying between $a\,(100)$ and $m\,(110)$. Suppose, however, we write $\frac{1}{y}$ for y in the equation to the ray, we obtain

$$xy = \tan\theta,$$

i.e. a rectangular hyperbola, the asymptotes of which are the coordinate axes. This curve corresponds exactly to the ray and may be used in precisely the same way; the only difference is that the ratio of the indices has been inverted, and we now obtain the indices of faces lying between $m\,(110)$ and $b\,(010)$ without passing beyond the boundary $y = 1$.

In the same way we may see that the moriogram need not extend beyond the boundary $x = 1$. The effect of writing $\frac{1}{x}$ for x, which we remember represents the tangent of some angle, is to give us the cotangent of the same angle. The hyperbola now becomes the ray

$$x = y\cot\theta = y\tan(90^\circ - \theta),$$

which may be termed the complement of the previous ray.

Hence finally we perceive that the moriogram is bounded by a square, a side of which is the unit distance. A pencil of rays, one for every degree in the right angle, radiates from one corner, every fifth ray being drawn darker. Except in the case of the ray bisecting the right angle the intersections of the complementary rays with the opposite sides of the square are connected in pairs by rectangular hyperbolae, every fifth of which is also drawn darker. Traversing the moriogram are series of vertical and horizontal lines. The former lines cut the side which has above been termed the axis of X, such that the abscissae represent the tangents of some angle; again every fifth line is drawn darker. The horizontal lines divide the sides of the square at right angles to them into all fractional parts not involving numbers greater than 10. To avoid too great complexity, certain of the lines, namely those corresponding to the parts with the numbers 7 and 9 as denominators, have been drawn fainter. If the tenths of divisions be estimated we may read to $6'$; but for most ordinary purposes accuracy to $30'$ would be quite sufficient.

It may be noted that the known angle need not necessarily be *am* (100 : 110); it may just as conveniently be the angle between *a* (100) and any other face—except, of course, *b* (010)—lying in the zone.

As an example of the use of the moriogram, the readings observed for the principal zones of cerussite are given and compared with the values obtained by calculation.

Three Principal Zones of Cerussite.

	Measured from (100).	
	Observed.	Calculated.
(530)	20° 5′	20° 6′
(110)	*	31 23
(350)	45 30	45 28½
(120)	50 40	50 39½
(130)	61 25	61 25½
(180)	78 20	78 25

	Measured from (001).	
	Observed.	Calculated.
(105)	13°15′	13°20′
(104)	16 25	16 30½
(103)	21 25	21 33½
(102)	30 35	30 39
(101)	*	49 51
(302)	60 45	60 39
(201)	67 15	67 7½

	Measured from (010).	
	Observed.	Calculated.
(0.10.1)	7°50′	7°52½′
(091)	8 35	8 44
(081)	9 55	9 48½
(071)	11 5	11 10½
(061)	13 0	12 59
(051)	15 25	15 27½
(041)	19 5	19 4½
(031)	24 40	24 45
(052)	28 55	28 57
(021)	34 35	34 40
(032)	42 40	42 41
(076)	49 50	49 51
(087)	50 25	50 26
(011)	*	54 8
(023)	64 15	64 16
(012)	70 10	70 7½
(013)	76 30	76 27
(016)	83 5	83 8

The moriogram may, though not so readily, be used in cases where no right angles are available. Thus suppose the angles *ab* (100 : 010) and *am* (100 : 110) are ψ_0 and ψ_1 respectively. The course of procedure may be seen from the following equations:—

$$\frac{\cot\psi_1-\cot\psi_0}{y}=\frac{\cot\theta_1}{y}=\frac{1}{x}=\cot\theta=\cot\psi-\cot\psi_0.$$

Having found θ_1 we proceed in precisely the same way as before in the case where ψ_0 was a right angle, to find the values of θ corresponding to the indices of the faces in question.

The derivation of θ_1 from ψ_1 or of ψ from θ may be made graphically from the moriogram. The intercepts made on any vertical line by the rays of the pencil are, when measured from the axis of X, i. e. the lower side of the square, directly proportional to the cotangents of the corresponding angles. By means of dividers we can subtract $\cot\psi_0$ from $\cot\psi_1$. A convenient method is to use strips of paper divided on the edges exactly as the vertical lines in the moriogram corresponding to the angles 5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°; or perhaps better still to use a square sheet cut slightly larger than the square boundary of the moriogram, the eight edges, four on either side, being divided as the lines mentioned, omitting the first. Selecting the most convenient of these lines, i. e. the one furthest from the origin which meets all the rays in question within the limits of the moriogram, we place the point marked 90° on one scale on that marked ψ_0 on the other, and convert immediately from θ to ψ, or vice versa. The author has found it on the whole more convenient to keep this conversion distinct from the moriogram itself, and employs the following method:—four scales of cotangents (cot 45° being represented by 20, 10, 5, and 2 cm. respectively) are drawn on a sheet of paper and four scales identical with the above are drawn, two on either side, on a strip of paper; the procedure is the same as before.

As an example the readings obtained for the zone [100,001] of epidote are added and compared with the computed values:—

Zone [100,001] *of Epidote.*

	Observed.		Calculated.	
	Moriogram.	Corrected.		
$(\bar{3}01)$	14°45′	16°45′	16°46′	
$(\bar{2}01)$	21 40	25 55	25 57	
$(\bar{3}02)$	27 55	35 10	35 4½	
$(\bar{4}03)$	30 50	39 40	39 32	
$(\bar{1}01)$	38 25	51 40	51 41	Measured from $(\bar{1}00)$.
$(\bar{3}04)$	46 35	64 40	64 38½	
$(\bar{2}03)$	49 55	69 45	69 46	
$(\bar{1}02)$	57 50	81 0	81 2	
$(\bar{1}03)$	67 20	93 10	93 2	
$(\bar{1}04)$	72 30	99 1	99 0	
(001)	90 0	*	64 37	
(105)	75 55	54 0	53 58	
(102)	57 50	42 15	42 6	
(101)	38 25	*	29 54	Measured from (100).
(201)	21 40	18 15	18 25	
(301)	14 45	13 0	13 11	

To draw the curves, their intercepts on every ray of the pencil were calculated, and the points thus determined were pricked off on the diagram. Since from the equation

$$xy = \tan\theta,$$

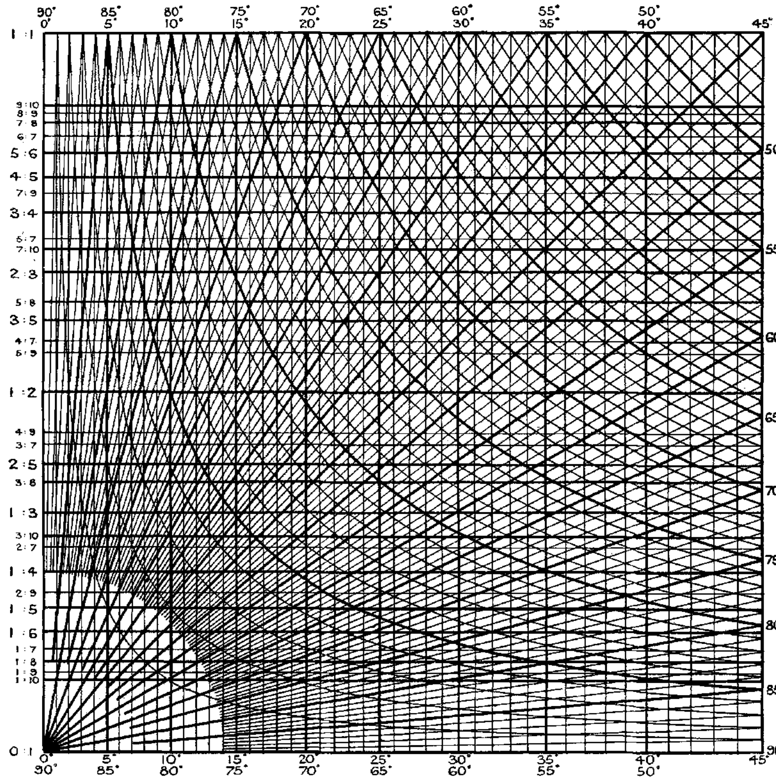
we have

$$\frac{dy}{dx} = -\frac{y}{x} = -\tan\phi = \tan(180^\circ - \phi),$$

where $y = r\sin\phi$, $x = r\cos\phi$, r and ϕ being radial coordinates, we see that the tangents to the curves at all points lying on the same ray are parallel, and that the ray and any tangent make supplementary angles with the same coordinate axis. The curves were developed by means of the tangents. Their direction for points lying on the same ray were conveniently found by means of the form of protractor[1] devised by the author for plotting gnomonic projections, and they were drawn for each curve in succession by sliding the protractor along a straight-edge. Finally, for the inking of these curves the templates[2] devised by Professor R. H. Smith were employed.

[1] Min. Mag., 1903, vol. xiii, p. 313; Zeits. Kryst. Min., 1904, vol. xxxix, p. 146.

[2] Set of 'R. H. S. Curves,' by Professor Robert H. Smith, published by Cassell & Co., London.



G. F. HERBERT SMITH: THE MORIOGRAM.