On some new forms of Quartz-wedge and their uses.

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THE determination of the amount and sign of the relative retardation of the wave-fronts of the two rays, into which a plane-polarized ray of light is resolved in passing through an anisotropic crystal, furnishes us with the readiest means of identifying transparent minerals occurring in thin sections or as minute fragments without definite crystalline form. In sections which are of practically the same thickness the individual crystals of each mineral show a certain range of colours resulting from the interference of the rays after resolution parallel to the principal plane of the analyser, and these colours are usually sufficient, with the help of other characters, for the identification of the mineral species. But when the thickness of the micro-sections varies considerably, or we are dealing with small grains from river sand or fragments resulting from the crushing of a rock, the problem is less simple, and quantitative determinations are required.

Every ray passing through a substance suffers retardation, that is to say, travels a less distance than it would traverse in the same time in vacuo. The retardation r_1 suffered by a ray of light in traversing a unit of distance in any isotropic substance is $\frac{v}{v_1}-1$, where v is the velocity in vacuo and v_1 the velocity in the substance, for $\frac{v}{v_1}$ is the distance the light would have gone in vacuo in the same time as it occupied in traversing one unit of distance in the substance.

But $\frac{v}{v_1}=\frac{1}{u_1}=\mu_1$, where u_1 is the velocity in the substance, taking the velocity in vacuo as unity, and μ_1 is the index of refraction in the substance. Accordingly, we have

$$r_1=\frac{v}{v_1}-1=\frac{1}{u_1}-1=\mu_1-1.$$

The quantity r_1, the retardation in a unit of length, may be termed the index of retardation.

In the case of light, travelling in a given direction, entering an anisotropic substance it is resolved in general into two rays propagated at different rates. If the wave-fronts of these rays have velocities v_1 and v_2, or, taking the velocity in vacuo as unity, u_1 and u_2, and their indices of refraction are μ_1 and μ_2, and indices of retardation r_1 and r_2,

$$r_1 = \frac{v}{v_1} - 1 = \frac{1}{u_1} - 1 = \mu_1 - 1,$$

and

$$r_2 = \frac{v}{v_2} \quad 1 = \frac{1}{u_2} - 1 = \mu_2 - 1.$$

If d be the difference in the retardation of the two wave-fronts in a unit of length parallel to the wave-normals[1],

$$d = r_1 - r_2 = \frac{1}{u_1} - \frac{1}{u_2} = \mu_1 - \mu_2.$$

The quantity d is usually known as the strength of double refraction, or birefringence, but it is better described as the relative retardation in a unit of length, or the index of relative retardation.

If, instead of a unit length, a distance l be traversed parallel to the wave-normals in the anisotropic medium and k be the total relative retardation of the two wave-fronts in passing through that distance, we have

$$k = ld.$$

For instance, if a thin section of quartz be cut parallel to the axis, the index of relative retardation (birefringence) of light traversing it approximately at right angles will be 0·0091 (for sodium-light): then if the thickness of the section be 0·023 mm. we have

$$k = 0{\cdot}023 \times 0{\cdot}0091 = 0{\cdot}000209.$$

It is, however, usual to take as the unit of relative retardation a micromillimetre, or millionth part of a millimetre, as by so doing numerous decimal places are avoided. This unit is, however, too small for measuring the thickness of thin plates and small crystals, and I have found it convenient to take as unit of thickness a thousandth part of a millimetre, viz. a thousand micromillimetres. The relative retardation in micromillimetres in traversing this distance will be a thousand times that in traversing a micromillimetre, that is, a thousand times the absolute index of relative retardation or birefringence; and it is free from the disadvantage of being a small fraction of unity.

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[1] Strictly speaking the wave-fronts will be parallel only if the ray meets a plane surfaceof the anisotropic medium at right angles.

If then

K = relative retardation in micromillimetres,

L = thickness in thousandths of a millimetre,

D = relative retardation in micromillimetres in traversing one thousandth part of a millimetre

$$= 1000\,d = 1000\left(\frac{1}{u_1} - \frac{1}{u_2}\right) = 1000\,(\mu_1 - \mu_2),$$

we have

$$K = LD.$$

D may be considered a working index of relative retardation one thousand times the absolute index. The calculation given above then becomes

$$K = 23 \times 9{\cdot}1 = 209.$$

To ascertain the index of relative retardation in any particular direction in a crystal it is, therefore, sufficient to know the thickness of the crystal in the direction in question and the relative retardation that takes place in that distance[1]. The latter is approximately determined by reference to a scale of Newton's colours, either in a coloured plate or expressed in words. In neither case are very satisfactory results obtained, especially when the colours become fainter or entirely disappear as the relative retardation increases. By the use, however, of the quartz-wedge inserted in the position of compensation[2] it is possible, when the relative retardation is not too great, to determine to which of the orders in Newton's scale the colour belongs.

To secure more exact determinations of the relative retardation various means have been employed, among which may be mentioned the bifractometer, the Babinet compensator, and the twin-compensator. These are, however, complicated pieces of apparatus, designed rather for the determination of physical constants than for the identification of small mineral particles.

In the laboratory of the Scientific and Technical Department of the Imperial Institute it is frequently necessary to recognize small sand-grains and fragments of crushed rock or veinstuff, and for this purpose a quartz-wedge, that has been made under my directions by Messrs. R. & J. Beck, Ltd., has proved very useful.

Its angle is much larger than that of wedges ordinarily in use, the maximum thickness being as much as a millimetre and a half. It is

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[1] See Mallard, Traité de Cristallographie, Paris, 1884, vol. ii, p. 399.

[2] For this purpose it is best placed in the focus of the eye-piece, a microscope being employed in which the analyser is above the eye-piece.

graduated so as to show the relative retardation of light passing through it at different points, the relative retardation at two adjoining divisions differing by a thousand micromillimetres. The divisions were fixed by ascertaining with a micrometer-screw the points at which the thickness amounted to multiples of $\frac{1000}{9 \cdot 1}$ thousandths of a millimetre[1]. I calibrated the wedge by placing it in sodium-light, first between crossed nicols, when the dark bands gave the points where the relative retardation amounted to an even number of semi-wave lengths, and then between parallel nicols, when similar bands gave the points where it amounted to an odd number of semi-wave lengths. The average difference of relative retardation between adjacent lines was thus calculated to be 999·6 micromillimetres.

With this wedge relative retardations up to 14,000 units (about twenty-eight 'orders' of colours) could be observed. A still thicker wedge is, however, sometimes needed.

In determining the index of relative retardation in small fragments they are immersed in a liquid of sufficiently high index of refraction to render them fairly transparent. The thickness is then determined by using a high power and focusing with the fine adjustment, which should be graduated in thousandths of a millimetre, first on an edge or crack on the top of the fragment and then on a scratch in the glass-slip on which it lies. This apparent thickness multiplied by the index of refraction of the liquid gives the real thickness of the fragment[2].

The relative retardation of light traversing the fragment is measured by inserting the wedge in the position of compensation till the dark band representing complete compensation is reached. The relative retardation is then read off directly from the wedge, amounts of less than a thousand micromillimetres being estimated from the position between the two nearest graduations.

If the surface of the fragment is irregular, coloured bands are seen, which are mainly due to the variation in the thickness of the fragment.

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[1] The graduations of the 'birefractometer' represent the thickness in decimal parts of a millimetre, not the relative retardation.

[2] The focus is best determined by using a strong monochromatic light passing through a small aperture six inches at least from the mirror of the microscope. Interference rings are formed unless the focus is very exact (see Lévy and Lacroix, 'Les Minéraux des Roches,' Paris, 1888, p. 60). The thickness may also be measured through the mineral and multiplied by its approximate index of refraction as estimated by Becke's method. This usually gives a sufficiently approximate value for the index of relative retardation to enable the mineral to be identified.

Some particular point in the crystal must accordingly be chosen for the determination both of the thickness and relative retardation.

If the index of relative retardation of the crystal is very high the bands are sometimes so narrow that it is difficult to distinguish the colours, and these are often appreciably modified by the variations in the index for different parts of the spectrum. An approximate result can then be obtained by observing the reading on the wedge when the bands begin to be visible in the part of the fragment selected for the observation, and the reading when they disappear; the mean of these readings will give with close approximation the relative retardation.

If a high power be used in determining the relative retardation the cone of light should be narrowed either by lowering the condenser or by the use of a diaphragm, so that the colours or bands may be more clearly visible.

For the determination of the index of relative retardation of minerals in thin sections, which show colours corresponding to comparatively low relative retardation, I find it convenient to use a double quartz-wedge[1], which I have also had made by Messrs. R. & J. Beck, Ltd., for the Imperial Institute. It consists of two quartz-wedges placed close together, side by side. One is cut in the usual manner with its length parallel to the vertical axis of the crystal, the other with its breadth parallel to the same direction, so that when placed with their lengths parallel they have their axes at right angles and therefore extinguish simultaneously between crossed nicols. They are ground down together to the same slope, and when placed obliquely between crossed or parallel nicols they show exactly the same bands of colour, which pass continuously across both wedges. If now a crystal-plate is also placed in the position of extinction between the crossed or parallel nicols, the two wedges continue to show the same colours; but if, by the rotation of the crystal or of the nicols and double wedge together, the directions of extinction of the crystal are parallel to those of the double quartz-wedge and make an angle of 45° with the nicols, one of the wedges will show a black band indicating the position of compensation, while the second will show colours corresponding to increased relative retardation. It is evident that the colour, which on the latter wedge is opposite the centre of the dark band of the former, denotes a relative retardation exactly double that of the crystal-plate alone, and thus a more accurate determination of the relative retardation of the plate may be made. Either the relative

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[1] I find that somewhat the same principle has been applied in a different manner in the twin-compensator.

retardation corresponding to the colour opposite to the centre of the dark band should be divided by two; or, better, the colour on one wedge opposite to a sensitive tint (*teinte sensible*) on the other should be noted, and the difference of the relative retardations corresponding to these colours, or the sum of those relative retardations if one of the colours is between the dark band and the thin end of the wedge, should be halved in like manner. Additional accuracy can be obtained by making several similar observations with crossed and parallel nicols. With low relative retardations parallel nicols usually show more distinctive tints.

The double wedge is also more convenient than the single wedge for determining the sign of a direction of extinction in a mineral in a thin section, for a comparison of the two sides tells clearly and at once which shows compensation and which addition.

It can also be used as a stauroscope; for, if a crystal-plate is rotated between crossed or parallel nicols with the double wedge at 45° to them, or the nicols and wedge are rotated together, the crystal being stationary, the position of extinction is known by the exact correspondence of the bands on the two wedges.

It is scarcely necessary to add that, as pointed out by Mallard and by Lévy and Lacroix[1], the thickness of a section may be determined by observing which of a number of individuals of some known mineral has the colour representing the greatest amount of relative retardation. It may then be assumed that this individual is cut so that the velocities of the two wave-fronts of light propagated transversely to the section are approximately the same as the greatest and least velocities in the mineral, and that the index of relative retardation at right angles to the section is nearly equal to the maximum index of relative retardation in the crystal; the latter being the difference between the greatest and least retardations, or, what is the same thing, the difference between the greatest and least indices of refraction—or a thousand times this amount if the working index of relative retardation, which I have suggested, is employed in place of the absolute index.

We can now calculate the thickness of the section at once from the formula $K = LD$, where K is the observed relative retardation in millimetres, $D = 1000\,(\gamma - \alpha) = 1000\left(\frac{1}{u_\gamma} - \frac{1}{u_\alpha}\right)$, and L is the thickness in thousandth parts of a millimetre.

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[1] loc. cit.