A simple arrangement and notation of the thirty-two classes of symmetry based on the symmetry of zone-axes.

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IF a straight line making an angle with a zone-axis of a crystal be rotated about the zone-axis while the crystal remain fixed, the physical characters of that line will pass through a cycle of changes. If these changes repeat themselves n times during the course of a complete revolution, the zone-axis is said to have an nth-turn or n-fold cyclic symmetry. If the line be at any time normal to a crystal-face it will be normal to a face with the same physical characters n times during such a revolution.

If the order of succession of the changes be dependent on the sign of the direction of rotation, the zone-axis is said to be 'unilateral' or 'haplo-cyclic': if the order be the same for opposite directions of rotation, it is 'bilateral' or 'amphi-cyclic'[1].

The symmetry *round* a zone-axis, or its cyclic symmetry, may therefore be expressed by the cyclic number of the zone-axis, which may be I, II, IV, III, or VI, and its character whether unilateral (U) or bilateral (B).

A zone-axis has two opposite longitudinal directions. The termination corresponding to one of these may be defined as the aggregate of all straight lines making an angle of less than a right angle with such direction—as well as the faces to which any of such lines are normal. The two terminations of a zone-axis may be expressed by means of the zone symbols thus: T [uvw] and T [$\bar{u}\bar{v}\bar{w}$].

The symmetry *along* a zone-axis, or its 'longitudinal' symmetry, expresses the relation between its two terminations.

If there be no symmetry connecting the two terminations, other than symmetry round the zone-axis, the terminations will be unlike and the

[1] The number of the planes of symmetry passing through a bilateral zone-axis is that of its cyclic symmetry. If this number be odd, they are all like; if it be even, they fall into two equal groups of unlike planes. A unilateral zone-axis has no planes of symmetry passing through it.

zone-axis is said to be uniterminal (u). In this case, if the zone-axis be unilateral, the crystal and its reflexion are not superposable.

If the two terminations of a zone-axis are similar it is said to be biterminal. In such cases the terminations may be related in different ways.

If one termination be the reflexion of the other in a plane of symmetry at right angles to the zone-axis, the longitudinal symmetry is said to be reflected (r).

If one termination can be derived from the other by reflexion and a half turn round the zone-axis, the axis is said to have 'longitudinal half turn' or 'central' symmetry (c). The crystal has symmetry about a point, and opposite directions on any straight line have the same physical characters. If the number of the cyclic symmetry be even, reflected and central symmetry are identical.

If one termination be derived from the other by reflexion and a quarter turn, the axis has 'longitudinal quarter turn' symmetry (q). It can be easily shown that this only gives rise to new forms of symmetry if the number of cyclic symmetry be II; in other words, if there be a half turn symmetry round the zone-axis. It can be also shown that longitudinal one-third turn and one-sixth turn symmetries give no new forms.

In all cases where the longitudinal symmetry of a unilateral zone-axis involves reflexion with or without rotation the two terminations are not superposable, but the crystal and its reflexion are superposable.

One termination may also be derived from the other by rotation through a half circle on a zone-axis at right angles to the original zone-axis. If the symmetry round the axis be bilateral, the relation between the terminations will be the same as in the case of central symmetry. New forms of symmetry are therefore only obtained when the cyclic symmetry is unilateral. In such cases the longitudinal symmetry may be referred to as 'helical' (h), because the two terminations are related in the same manner as the ends of a screw[1]. The terminations are superposable, but a crystal and its reflexion are not superposable.

[1] The number of secondary axes will be equal to that of the cyclic symmetry of the original axis. If that be odd, they will be all alike and uniterminal. If it be even, there will be two equal groups of unlike helical axes.

If a straight line at right angles to a helical zone-axis be rotated about it, the order of succession of changes of physical characters will be the same for opposite directions of rotation, except that the relation of such characters to opposite ends of the zone-axis will be reversed. In like manner and subject to a similar proviso, the faces of a helical zone exhibit the same succession in opposite directions. These relations do not exist in the case of other unilateral zone-axes.

The full symmetry of a zone-axis may therefore be expressed by the conjunction of the different kinds of symmetry which it exhibits, and be indicated by the association of their symbols; thus the symmetry of the principal axis of idocrase may be described as 'fourfold' or 'quarter turn' 'bilateral central', or it may be more briefly and conveniently referred to by the symbol IVBc. If desired for the sake of brevity, the words 'whole turn' and 'unilateral' may be omitted, thus the symmetry of the principal axis of scapolite or scheelite may be described as 'quarter turn unilateral central' or simply 'quarter turn central', but the full symbol IVUc should always be used. The symmetry of a zone-axis of crystals of the asymmetric, calcium thiosulphate, class would be 'whole turn unilateral uniterminal' or simply 'uniterminal,' with the symbol IUu.

The thirty-two classes of crystal-symmetry may be distinguished by the symmetry of the zone-axis having the highest number of cyclic symmetry and may be referred to by the corresponding symbols. Those classes which have four axes of threefold or one-third turn cyclic symmetry are said to be 'cubic' with the symbol 4III or C.

The classes are first grouped by the number of the characteristic cyclic symmetry, the cubic classes forming a separate group. These groups are subdivided, first, according as the cyclic symmetry is bilateral and unilateral, and secondly, by the character of the longitudinal symmetry.

In the first table which accompanies this paper each class is represented by its symbol accompanied by the name of a substance crystallizing in it, when such is known to exist. The classes with the same number of cyclic symmetry are arranged vertically, and those that agree in their unilateral or bilateral character and longitudinal symmetry horizontally[1]. It will be seen that all the vertical columns except that of the whole turn symmetry have *unilateral* uniterminal, helical, and central symmetry and *bilateral* uniterminal and central symmetry. These give $5 \times 5 = 25$ classes; the whole turn column has three classes, and the half turn column and one-third turn column have each two extra classes, making the full total of thirty-two classes.

In the second table, each symbol is followed by the crystallographic system and the number of the class according to Professor Lewis ('Crystallography,' 1899) and Professor Miers ('Mineralogy,' 1902, p. 280).

This classification presents many points of resemblance to those of

[1] It is convenient to arrange the different kinds of longitudinal symmetry in an order different from that in which they have been described.

TABLE I.

	Whole turn. I.	Half turn. II.	Quarter turn. IV.	One-third turn. III.	One-sixth turn. VI.	Four one-third turns or Cubic. 4III or C.
UNILATERAL (U).						
Uniterminal (u).	IUu Calcium thiosulphate.	IIUu Cane sugar.	IVUu Wulfenite.	IIIUu Sodium periodate.	VIUu Nepheline.	CUu Ullmannite.
Biterminal						
Helical (h).	(*See IIUu*)	IIUh Sulphur.	IVUh Strychnine sulphate.	IIIUh Quartz.	VIUh Barium antimonyl dextro-tartrate + potassium nitrate.	CUh Cuprite.
Central (c).	IUc Albite.	IIUc Augite.	IVUc Scapolite.	IIIUc Dioptase.	VIUc Apatite.	CUc Iron-pyrites.
Reflected (r).	(*See IBu*)	—	—	IIIUr (Unknown).	—	—
Quarter turn (q).	—	IIUq (Unknown).	—	—	—	—
BILATERAL (B).						
Uniterminal (u).	IBu Clinohedrite.	IIBu Hemimorphite.	IVBu Succin-iodimide.	IIIBu Tourmaline.	VIBu Iodyrite.	CBu Tetrahedrite.
Biterminal						
Central (c).	(*See IIUc*)	IIBc Olivine.	IVBc Idocrase.	IIIBc Calcite.	VIBc Beryl.	CBc Galena.
Reflected (r).	(*See IIBu*)	—	—	IIIBr (Unknown).	—	—
Quarter turn (q).	—	IIBq Copper-pyrites.	—	—	—	—

TABLE II.

Symbol.	System.	Lewis.	Miers.	Symbol.	System.	Lewis.	Miers.
IUu	Triclinic	I	1	IIIUu	Rhombohedral	I	9
IUc	,,	II	2	IIIUh	,,	IV	10
IBu	Monoclinic	II	3	IIIUc	,,	II	15
IIUu	,,	I	4	IIIUr	,,	VI	11
IIUh	Orthorhombic	I	6	IIIBu	,,	V	12
IIUc	Monoclinic	III	5	IIIBc	,,	III	19
IIUq	Tetragonal	VII	22	IIIBr	,,	VII	13
IIBu	Orthorhombic	III	7	VIUu	Hexagonal	I	14
IIBc	,,	II	8	VIUh	,,	V	16
IIBq	Tetragonal	I	26	VIUc	,,	II	17
IVUu	,,	III	21	VIBu	,,	III	18
IVUh	,,	V	23	VIBc	,,	IV	20
IVUc	,,	IV	24	CUu	Cubic	III	28
IVBu	,,	VI	25	CUh	,,	I	29
IVBc	,,	II	27	CUc	,,	IV	30
				CBu	,,	V	31
				CBc	,,	II	32

Professor Miers and Mr. H. Hilton[1]. The latter I had not seen when I read this paper. I recognize the justness of his contention that a centre of symmetry is more fundamental than a plane of symmetry. The chief merit I claim for the present arrangement is that it will be easily apprehended, remembered, and applied by the student. If thought fit Professor Miers's terms 'digonal,' 'tetragonal,' 'trigonal,' 'hexagonal,' and 'tesseral' may be substituted for II-, IV-, III-, and VI-turn, and cubic respectively, and 'di-' prefixed to indicate a bilateral character of the axis. The only difficulty is that some of these expressions have already been widely used in other senses.

By far the most convenient method of referring either viva voce or in writing to any class will be found to be its symbol. Thus quartz may be said to crystallize in IIIUh class; iron-pyrites in the CUc (Cubic Uc) class.

[1] Min. Mag., 1907, vol. xiv, p. 261.