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On a new method for studying the optical properties of crystals.[1]

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BEFORE treating of the subject before me, I must describe its history. At a soirée of the Royal Society on April 25, 1877, I showed a number of specimens illustrating a new class of optical properties, applicable to the identification of minerals. These greatly

[1] The MS. of this paper was finally prepared for press by Dr. Sorby during his last illness. It contains the details and complete account of his work, of which preliminary accounts were published in the first two volumes of the Mineralogical Magazine under the following titles :—

'On a simple method of determining the index of refraction of small portions of transparent minerals.' 1877, vol. i, pp. 97–98.

Presidential address. [Determination of refractive indices in mineral plates, &c.] 1877, vol. i, pp. 193–208, with plate VII.

'On the determination of minerals in thin sections of rocks by means of their indices of refraction.' 1878, vol. ii, pp. 1–4.

'Further improvements in studying the optical characters of minerals.' 1878, vol. ii, pp. 103–105.

The method he describes for determining refractive indices is identical in principle, though worked out in far greater detail, with that given by the Duc de Chaulnes in 1767 for singly-refracting substances ; but, as may be gathered from a remark in the last of the papers quoted above, the method was devised quite independently by Dr. Sorby.

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An obituary notice, with portrait, of Dr. Sorby appeared in the last number of this magazine (vol. xv, p. 180).

interested the late Sir G. G. Stokes, Bart., and led to a considerable amount of correspondence between us; and he communicated a paper to the Royal Society on the mathematical part of the subject,[1] and I sent a short one [2] on my apparatus and observations. This was admitted to be very imperfect, since the crystals examined, though showing the general facts perfectly well, had not been cut so as to be suitable for such quantitative measurements as could be compared with Stokes's theoretical determinations. In my short paper I therefore said that I intended to prepare a variety of crystals, accurately cut in suitable directions, and to make use of fresh means to correct sundry small instrumental errors. All this I carefully carried out, and wrote a paper almost finished and labelled 'Manuscript for the Royal Society'. I then left home for some time, and a number of important circumstances turned up to draw away my attention, and it was not till after thirty years that I was astonished to find the manuscript.

It is now so long since the papers by Stokes and myself were published, that it seems desirable to describe the older and newer apparatus employed, added to a first-class Smith & Beck's microscope. These are as follows:—A scale and a vernier attached to the stand and body of the microscope, so that the up and down movement may be measured to $\frac{1}{1000}$th inch. Underneath the stage a glass plate with two sets of lines $\frac{1}{100}$th of an inch apart, ruled at right angles to one another, brought to focus at about the level of the stage by the achromatic condenser. Close to the glass plate is an iris-diaphragm, so that a circular hole of variable size can be brought to focus along with the ruled lines. Below can be inserted or withdrawn a Nicol's prism, capable of rotation. A cap fitting on the object-glass with a small hole to reduce the residual spherical aberration; and another cap to cut off half the area of the object-glass in any desired direction, to detect and determine the plane of polarization of any beam that has passed through the crystal under examination.

These simple appliances open out a wide field for study in connexion with mineralogy. To quote from Stokes's paper: before the time of Fresnel it was thought that one of the rays in a biaxial crystal obeyed the ordinary laws of refraction. To prove that this was not the case required skill on the part of the optician who worked the crystal. 'It is interesting to find that the extraordinary character of the refraction of

[1] G. G. Stokes, 'On the foci of lines seen through a crystalline plate.' Proc. Roy. Soc. London, 1877, vol. xxvi, pp. 386–401.

[2] H. C. Sorby, 'On some hitherto undescribed optical properties of doubly refracting crystals.—Preliminary notice.' Ibid., pp. 384–386.

both rays in a biaxial crystal admits of being established by such comparatively simple methods of observation as those adopted by Mr. Sorby.'

In the following original paper my chief aim was to show that my measurements and the general facts agree so closely with Stokes's mathematical calculations as to prove that all are substantially correct, though there may be a few small residual errors. Though tempted to add to the paper, it may be well to leave it nearly as it was, as being more in accordance with what had met with Stokes's approval. *The following is the paper written thirty years since* :—

Determination of the True Indices.

Professor Stokes's theoretical deductions and my own observations clearly prove the importance of distinguishing between real and apparent indices, and show in what circumstances, and in what manner, the true indices may be determined. There is no difficulty whatever in the case of unifocal images, since the apparent index is the true index (μ), no matter what may be the direction in which the section is cut; but in the case of bifocal images the connexion between them is somewhat complex, unless the section is in a suitable direction, and even then some calculation may be necessary. The true index (μ') of the single extraordinary ray of crystals having only one bifocal image cannot be conveniently determined unless the section be either parallel or perpendicular to the axis, in which latter case the index (μ) of the ordinary ray must be known; the two images being distinguished by using the semicircular stop in front of the object-glass, already explained. If A be the apparent index, μ' can be calculated from the equation $\mu' = \sqrt{A\,\mu}$. When the section is parallel to the axis, the apparent index (a) for lines parallel to the axis of the crystal and perpendicular to the plane of polarization is the true index (μ') of the extraordinary ray, which may also be calculated from the other apparent index (b) by means of the following equation, $\mu' = \dfrac{\mu^2}{b}$. We thus obtain its value by two different measurements, the mean of which may be adopted, unless there be some doubt as to the section being parallel to the axis, since a slight inclination would reduce the value calculated from b much more than that determined directly from a. I give examples of the application of these methods.

If the crystal has two bifocal images, polarized in opposite planes, giving the four apparent indices $\frac{a}{b}\,\frac{c}{d}$, the true values of the three indices of refraction, μ, μ', μ'', may be either directly observed or calculated, if the section be in the plane of any two of the three axes of the crystal. In this case a, b, c, d become respectively $\frac{\mu''^2}{\mu'}$, μ', μ, $\frac{\mu''^2}{\mu}$.

In each of these images the apparent index for lines perpendicular to the plane of polarization (b) and (c) is a true index, and the value of μ''^2 may be determined from a and from d by means of the following equations, $\mu'' = \sqrt{a\,\mu'}$ or $\sqrt{d\,\mu}$.

If the section was accurately cut, and the observed values of a, b, c, and d strictly correct, we ought therefore to have $\frac{a}{d} = \frac{c}{b}$; but, since errors can be scarcely avoided, this relation may not be found to be exact. Assuming that the errors are equally distributed over all the four determinations, we may, however, so divide between a, b, c, and d the discrepancy thus found, as to obtain the corrected values a, b, c, d, in which the relation $\frac{a}{d} = \frac{c}{b}$ is strictly true. In this manner we make use of four different observations to obtain the most probable mean values of the three indices.

If a crystal having two bifocal images be cut in the plane of only one axis, only that one of the three indices which corresponds to this axis can be determined by any simple means. It is that which is observed from the system of lines parallel to the axis, in the image polarized in a plane perpendicular to it. If, however, the section be oblique to all the axes, the true value of none of the indices can be determined, either by direct observation or by any simple calculation.

As an example I give the corrected apparent indices in the case of two sections of aragonite, and the true indices deduced from them.

Perpendicular to the axis.

1·387	1·677
1·675	1·385

$\mu = 1{\cdot}677$

$\mu' = 1{\cdot}675$

$\mu'' = \sqrt{1{\cdot}387 \times 1{\cdot}675} = 1{\cdot}524$, or $\sqrt{1{\cdot}677 \times 1{\cdot}385} = 1{\cdot}524$.

Parallel to the axis, perpendicular to the twin-lamellae.

1·655	1·515
1·690	1·846

$\mu = 1{\cdot}690$

$\mu' = \sqrt{1{\cdot}655 \times 1{\cdot}690} = 1{\cdot}672$, or $\sqrt{1{\cdot}515 \times 1{\cdot}846} = 1{\cdot}676$.

$\mu'' = 1{\cdot}515$

Whence the means are:—

$$\mu = 1{\cdot}683,\ \mu' = 1{\cdot}674, \text{ and } \mu'' = 1{\cdot}520.$$

Measurement of the Indices of Refraction.

In order to determine the value of their indices of refraction with such an instrument as I have described, it is requisite to measure the thickness (t) of the substances, and the displacement (d) of the focal length, due to the light passing through them. The index of refraction of each specimen can then be calculated from the equation $\mu = \frac{t}{t-d}$. The thickness (t) of the specimen is determined by first adjusting the focus to particles of dust on the supporting glass, reading off the position on the scale, and then adjusting the focus for particles of dust on the upper surface of the mineral, and reading off this position. The difference in the readings gives the thickness. In order to determine the displacement of the focus when looking through the mineral I make use of a grating with two sets of parallel lines $\frac{1}{100}$th of an inch apart, ruled at right angles to one another on the same surface of a plate of glass, which is fixed some distance below an achromatic condenser with a small stop, and the image thrown just below the upper surface of the glass slip supporting the mineral. This is far better than having the lines on a plate placed on the stage, since we can move the object without altering the position of the lines. By this arrangement we see sharp and well-defined lines, that cannot be mistaken for any others. These of course are first viewed through the supporting glass alone, and afterwards through the mineral, the focus in both cases being carefully adjusted for the centre of the field. The decrease (d) in the focal length is given by the difference between the two sets of readings. If the upper surface of the mineral is covered with thin glass cemented with Canada-balsam, its thickness is measured, and, in calculating out the results, allowance is made for it in the following manner. The index of refraction of the covering glass combined with a thin layer of Canada-balsam, being about 1·53, the focal length is increased 0·53 for every 1·00 in the apparent thickness of the glass, as measured by looking through it. This is known from the difference between the focal points of dust on the upper surface and of small scratches or other marks on the surface of the mineral, and 0·53 of its value must be deducted from both t and d; the value of t being of course reckoned from the apparent upper surface of the mineral, and not from the top of the glass cover. Care must be taken to avoid any errors which might result from moving the stage of the microscope. In order to obtain strictly accurate measurements, all parts of the instrument should be well made, and a monocular arrangement used, since with a binocular the focal adjustment may not be the same for both images.

I have usually employed a No. 2 eye-piece and a $\frac{2}{3}$-inch object-glass, stopped down to an aperture of 13°, to avoid any effects of spherical aberration. A difference in adjustment of $\frac{1}{1000}$th of an inch is thus easily recognized. We must, however, not forget that the eye itself has some range in focal adjustment, and, even with every care, individual measurements may differ by $\frac{1}{1000}$th of an inch. The values of t and d might thus each vary by $\frac{2}{1000}$ths of an inch; and, if the variation were in opposite directions, $t-d$ might vary by as much as $\frac{4}{1000}$. This source of error may to a great extent be overcome by making a number of measurements and taking the mean, which then might usually not differ more than $\frac{1}{10000}$th of an inch from the truth. If the specimen examined be not less than $\frac{1}{4}$ inch thick, and in every other respect suitable for the purpose, the errors in the value of the index should be confined to the third place of decimals. Some of the specimens described in this paper are not, however, in every respect suitable for accurate measurements, and the errors in the *actual values* of the index may extend to the second place of decimals. For the general purposes of this paper this is of comparatively little importance, since the true value of the indices of different specimens of the same mineral may vary as much. The *relative values* of those measured in the *same* specimen are, however, dependent on *only two* sets of measurements, and therefore the chances of error are reduced to at least one-half. In some cases I think that the difference between the indices may be relied upon to another decimal point further than their actual values. I am anxious to call attention to this, since, though I look with suspicion on the absolute values of some of the indices given in this paper, I think that the relative values of those for any one single specimen are quite sufficiently accurate to establish the general principle of my subject.

In order to ascertain how far the method may be relied on, Professor

Stokes has determined the index of refraction of three different specimens of glass by the usual method, and I have determined them in the manner described above. The best specimen for the purpose was a light crown-glass. I took the mean of fifteen separate measurements. On using the light transmitted by blue glass, I found that the dark lines were scarcely visible. I also found that there was no sensible difference in the focal length when I used simple gas-light or the same transmitted through red glass which cuts off all rays more refrangible than D. This is perhaps because the focus-giving power of the red end of the spectrum of gas-light overpowers the effect of the blue end. However, to make sure I have in most cases used the red glass, and then the indices of refraction which I give represent those corresponding to about the centre of the red. I obtained for the index of this crown-glass 1·502. Professor Stokes obtained for the chief line of lithium 1·5011, and for that of sodium 1·5040. The mean of these would very well represent the centre of the red, and gives 1·5025, which thus differs very little from my result. In the case of the other glasses this same mean was found by him to be for one 1·6164, and for the other 1·8450. My results, deduced from only four or five separate measurements, were 1·620 and 1·838, one being thus rather too great and the other rather too little.

We therefore see that by eliminating the errors due to the slight range in focal adjustment, by taking the means of a number of measurements, a very satisfactory result may be obtained, and we also see that, when only a few observations are made, there may be a very decided error in excess or the reverse. These comparisons, however, show that the general principle is correct with suitable specimens, and that indices may be thus determined sufficiently accurately for the purpose to which I am about to apply them.

Nearly all the sections of the minerals examined were made by myself. Some of the calcite, aragonite, barytes, and fluor, were specially cut for the present purpose, but many of the rest were prepared long ago in connexion with various other subjects and were not all as good as could be desired. Except in a few cases the sections were cut more in relation to crystal forms than to the optic axes, and I have generally endeavoured to work them parallel to the commonest faces or well-marked cleavage, so that the results might be easily applied to the study of crystals in their natural state, or to portions obtained by cleavage. When I speak of the axes, I therefore generally refer to the crystallographic, and not to the optic, axes. It appeared to me desirable thus to adapt the subject to practical mineralogy, even though this made the results less suitable for comparison with Professor Stokes's theoretical deductions. I have also given the directions in which the sections are cut, rather than those along which the light passes, since, though the mean direction of the latter is usually perpendicular to the former, the special results of my method mainly depend upon slightly divergent rays being collected by an object-glass.

In studying the optical characters of minerals I have found it extremely useful to have an iris-diaphragm fixed some distance below the achromatic condenser, which can at pleasure be inserted or reversed without disturbing the grating. An image of a hole of any desired magnitude can thus be examined through the minerals. In some cases it is very necessary to have this hole extremely small, and to employ a comparatively high magnifying power. Small holes, made in thin black paper with a fine needle, may also be used with advantage below the condenser. It must be borne in mind that the separation and distortion of the images of the hole depend upon the thickness of the mineral, the direction of the section, the intensity of its double refraction, the size of the hole, and the magnifying power used. In describing the characters of the various minerals I shall first describe the varying appearances of such a hole, the principal of which are shown in the following illustration (p. 214), and then the phenomena observed with the rectangular grating.

Crystals with no Double Refraction.

In looking at the circular hole through crystals which have no double refraction, a simple, single hole is seen at one particular level and in perfect focus all round, as shown by fig. 1. The lines of the grating are all seen at one single focal point, no matter what may be the azimuth of the crystal. Such crystals are truly unifocal. The value of the index of refraction for the simple ordinary ray can easily be determined, but it may be well here to say that I have very strong reason to believe that slight differences in the chemical composition of different specimens of the same mineral give rise to corresponding differences in the value of the index. This appears to be the best explanation of the small but yet decided differences between some of my own measurements *inter se* and between them and the determinations of other experimenters by methods previously adopted. As illustrations of crystals having these characters I refer to fluor, leucite, and garnet. Thus, for two different specimens of fluor, I obtained 1·442 and 1·446. Previous observers by the old method give 1·433 and 1·436.

Crystals with one Axis of Double Refraction.

The phenomena vary much according to the inclination of the plane of the section to the principal axis of the crystal. Iceland-spar is of course the best type. My sections vary from about 0·3 to 0·4 inch in thickness.

Calcite. According to previous observers, the indices of refraction for medium red light are for the ordinary ray (μ) 1·656, and for the extraordinary (μ') 1·485.

1. *Section cut perpendicular to the principal axis.*—In looking at the circular hole, along the line of the axis, where there is no double refraction for perfectly parallel rays, I was very much surprised to find that there are two focal points, widely separated. On adjusting for one focus the circular hole is seen not in any way distorted and in perfect focus all round. It is, however, surrounded by a nebulous larger circle, which is really the image of the second circle seen out of focus, as shown by fig. 2. On altering the focal adjustment the true bright image expands and the nebulous circle contracts until we obtain a simple circle with no definite outline, and finally we come to the true focal point of the second image surrounded by the other circle seen out of focus. As far as the general appearances are concerned, the phenomena are just as though we were looking at two equal circular holes in black paper fixed opposite to one another on the opposite sides of a piece of glass.

When the section is cut exactly perpendicular to the axis, and not in any way inclined, so that the line of vision is directly along the axis, the two holes lie exactly one over the other, and in neither is the light polarized. In this position, without any of the special arrangements described below, it is therefore impossible to distinguish the ordinary from the extraordinary way. However, if the section be somewhat inclined, so that the images are seen separated, both are polarized, and the ordinary ray can be distinguished by its being polarized in the plane of the principal axis, which may be known by being that along which the holes are separated.

On examining the grating we see that there are two different focal points, wide apart, at which the two systems of lines at right angles to each other are in perfect focus at the same time, quite independently of their azimuth. These two images, one really and the other apparently unifocal, cannot be conveniently distinguished by inclining the section, since that alone is sufficient to make a true unifocal image appear bifocal. I am indebted to Professor Stokes for suggesting a method which enables us to ascertain which is due to the ordinary and which to the extraordinary ray. If a cap with a narrow slit be placed over the object-glass, so that the images are formed merely by a band of light extending over a diameter, that due to the ordinary ray is polarized in the plane of the slit, and that due to the extraordinary ray is polarized in the opposite plane. If this slit be too narrow, the objects are badly defined, and, if too broad, the two images are very imperfectly polarized. I find that with my $\frac{2}{3}$-inch object-glass a slit about $\frac{1}{50}$th of an inch wide gives a good medium result, since the definition is sufficiently good and the two beams sufficiently polarized to remove all doubt as to which is which. The slit should be arranged parallel to one of the systems of lines. That system cannot be seen at one definite focus, but along a strip passing through the centre of the field in the direction of the slit; the lines perpendicular to it are well defined at two different foci, and the pencils of light giving the images are polarized in the two opposite planes.

The disadvantages arising from the definition with the narrow slit being good only over a narrow band have induced me to contrive another method, which not only serves to distinguish the two images by the difference in their plane of polarization, but also clearly shows that one is not truly unifocal. I employ a cap over the object-glass, stopping off exactly one-half of the front lens, which thus has a semicircular aperture, the diameter being arranged parallel to one or other of the two sets of lines in the image of the grating. With such an object-glass the definition is not materially impaired, but the light passing somewhat obliquely through the mineral under examination gives rise to results which are to some extent as though the section were cut not quite perpendicular to the axis. In the case of the section now being described the ordinary ray is well polarized in a plane perpendicular to the edge of the semicircular stop, and the extraordinary ray in the opposite plane. The ordinary ray remains unifocal, whereas the extraordinary ray may be seen to have to a slight but yet decided extent the bifocal character more particularly described in the sequel.

The indices of refraction, measured in the manner described above, were found to be 1·659 for the ordinary, and 1·335 for the extraordinary ray, which, however, is only an apparent index. The true index of the extraordinary ray is 1·487, and according to Professor Stokes's theoretical deductions the above apparent index should be $\frac{(1{\cdot}487)^2}{1{\cdot}659} = 1{\cdot}333$.

This specimen was cut with very great care, and is free from every fault. Such a close agreement between theory and observation appears to me to prove very clearly that the general principles of both must be very nearly, if not absolutely, correct. Conversely, the index for the extraordinary ray, calculated from the above data, should be $1{\cdot}659 \times 1{\cdot}335 = 1{\cdot}488$.

2. *Section parallel to the cleavage.*—Looking at a hole previously arranged in the centre of the field of the microscope, it is seen to be widely separated into two as shown by fig. 3, one remaining in the centre, and the other thrown on one side somewhat distorted and apparently lying at a lower level. This difference between the ordinary and the extraordinary images is thus seen to very great advantage. In studying sections of minerals cut so as to show double refraction, it is, however, best to employ a Nicol's prism over the eye-piece, in order to examine separately the two rays polarized in opposite planes. No ordinary nicol, with sloping ends, should be placed between the condenser and the grating, since it makes the image to some extent bifocal. Arranging the analyser so that the plane of polarization is parallel to the principal axis of the crystal under examination, the central image of the circular hole, due to the ordinary ray, is seen to be well defined all round, and to have one simple and perfect focus. On the contrary, when the plane of polarization of the nicol is perpendicular to the principal axis, the displaced image, due to the extraordinary ray, is seen to be distorted in a remarkable manner. Its true size and shape cannot be seen at any adjustment of the focus, and on careful examination it is found that there is no true focus except for those parts of the circumference which are either parallel or perpendicular to the axis of the crystal, and that these lie in very different planes. If the hole be small, it is also seen to be much distorted. Adjusting the focus for those opposite sides of the circumference which are parallel to the axis, the hole is seen to be drawn out into a band in the line of the axis, having well-defined parallel sides, but ending in ill-defined extremities, one coloured blue and the other red. On drawing back the body of the microscope this band changes into a circular hole, several times larger than the real one, fringed on two sides with colour, and then changes into a band elongated in a direction perpendicular to the former. The sides are, however, never seen perfectly well defined, because they are fringed with colour, unless more or less perfectly monochromatic light is used for illumination.

On examining the grating when the nicol is so arranged that only the ordinary ray is transmitted to the eye, the image is seen to be truly unifocal. Both sets of lines are visible at one perfect focus, no matter what may be their azimuth, and the index of refraction is found to be

1·657, thus agreeing closely with the previous determination from a portion of the same specimen cut in another direction. On the contrary, when the nicol is so arranged that only the extraordinary ray passes, and the lines of the grating are parallel and perpendicular to the axis of the crystal, there are two widely separated focal points, at each of which *only one system* of lines can be seen. That which is parallel to the axis is well defined, and indicates an index of refraction of 1·412, whilst that system which is perpendicular to the axis is not seen as sharp lines, but as coloured bands, unless almost monochromatic red light be used, and then the apparent index of refraction is 1·578. The extraordinary image is therefore truly bifocal.

3. *Section cut nearly parallel to the principal axis.*—On examining a small circular hole we can see that neither image is displaced from the centre, but that they differ from one another most completely in other respects. Arranging the nicol so that only the ordinary ray passes to the eye, the circular hole is seen in perfect focus all round, and not at all modified in shape or size; but when the extraordinary ray passes to the eye, the hole is drawn out into a long band at two widely separated focal points as shown by figs. 4 *a* and 4 *b*, much in the same manner as previously described, only that it is elongated nearly twice as much as in the case of a section parallel to the cleavage, and almost entirely free from the coloured fringes. Judging from what is seen when the hole is small, a bright or dark point would, at two different and widely separated foci, be elongated into a line, which at one focus would be parallel to the axis, and at the other focus perpendicular to it, the elongation in the latter case being somewhat less than in the former. In intermediate positions we can see nothing but one or other of these lines more or less completely out of focus, perhaps so much so as to be practically invisible.

The facts just described make it easy to understand the peculiar phenomena observed when the grating is examined. Arranging the nicol so that only the ordinary ray passes to the eye, the two systems of lines are seen at the same focus independent of their azimuth, the image being thus perfectly unifocal. In the specimen examined the index of refraction was found to be 1·671. Many very careful measurements all agree, and I can explain this higher index only by supposing that the chemical composition of this specimen is not exactly the same as that of those previously described. In the case of the extraordinary image, the lines parallel to the axis give for the index 1·495, which exceeds the value of that for the extraordinary ray, deduced from the former specimen, by about the same amount as in the case of the ordinary ray. At all events we clearly see, that for lines parallel to the axis, the apparent index is the true index of the extraordinary ray. On the contrary, the apparent index for the lines perpendicular to the axis is no less than 1·851 or thereabouts. They are not perfectly well defined, and therefore it cannot be determined with any great accuracy. There is, however, no doubt whatever that it is far greater than that of the ordinary ray, since the lines are seen in focus when those due to the ordinary ray are completely invisible. According to Professor Stokes's theoretical deductions, it should be $\frac{(1{\cdot}671)^2}{1{\cdot}495} = 1{\cdot}868$. The difference between these determinations may safely be attributed to the fact of the section not being cut exactly parallel to the axis, having been made for general observation, when the leading facts were not understood, and not with the great care necessary to accurately verify this theoretical deduction. The error in direction would certainly have comparatively little effect on the value of the true index, deduced from lines parallel to the axis.

As previously named, the two systems of lines seen with the ordinary ray are perfectly distinct, no matter what may be their azimuth to the axis of the crystal. This is, however, not the case with those seen by means of the extraordinary ray. Having arranged them parallel and perpendicular to the axis, and having adjusted the focus to one or other system, on rotating the lines it will be seen that a very small change in the azimuth makes them appear broader and less distinct; and if, as in the specimens now being described, the spar be $\frac{3}{10}$ths of an inch in thickness, a still further change in the azimuth causes them to vanish entirely. The reason of this is very obvious. Each point in every line is drawn out into a line at two different foci, either in the direction of the axis or at right angles to it; and if these overlap, as they do when the lines are themselves in one or other of those directions, well defined lines are seen. On the contrary, if each point is drawn out in a direction considerably inclined to that of the lines, they are, as it were, spread over a surface many times greater than their true breadth, and so diluted with white light as to be completely invisible. It is to images having these characters that I apply the term *bifocal.*

It will thus be seen that this method of studying crystals shows in a most striking manner the difference between the ordinary and extraordinary rays. In the case of the ordinary ray, the phenomena differ in no way from what would be seen in looking through a piece of glass, or indeed in looking at a hole or grating without any intervening object. On the contrary, the phenomena seen by means of the extraordinary ray are most strikingly different, and one cannot help being at first greatly surprised to find that the two systems of lines ruled on exactly the same plane, seen through a perfectly transparent substance with parallel sides, lie as it were at two entirely different levels, and are visible only at one particular azimuth, and disappear on being rotated to other azimuths. Except that the light is polarized, this bifocal character of the extraordinary ray closely corresponds to the effects produced by inserting a cylindrical lens in front of the object-glass.

The relations of the above real and apparent indices will be better understood by collecting them together into a table, in which I give the best combined results of observation and theory.

Directions of the Sections.

Perpendicular to the axis.		Parallel to the cleavage.	Parallel to the axis.
1·658 (μ)	... ...	1·658 ...	1·658
1·332 $\left(\frac{\mu'^2}{\mu}\right)$	Line of axis ...	1·412 ...	1·486 (μ')
	Perpendicular to axis	1·578 ...	1·850 $\left(\frac{\mu^2}{\mu'}\right)$

According to this, the maximum difference between the foci in the bifocal image is 0·1324 of the thickness of the specimen under examination.

It will thus be seen that, whilst the index of the ordinary ray remains single and constant, that of the extraordinary ray, which is single and at a minimum in the case of the section perpendicular to the axis, breaks up into two, that for lines parallel to the axis gradually increasing until it becomes equal to the normal index, whilst that for lines perpendicular to the axis increases much more rapidly, until it attains the very high apparent index 1·850, which is about the same as the real index of dense flint-glass.

Other Minerals having one Axis of Double Refraction.

As examples of other minerals belonging to this division I would refer to parisite, proustite, zircon, and quartz.

Parisite (fluo-carbonate of cerium, lanthanum, and didymium). According to Senarmont, the index of refraction for the ordinary ray is 1·569, and for the extraordinary is 1·670; the double refraction being positive. My section, nearly perpendicular to the axis, shows extremely well two unifocal images, at two widely separated focal points, and gives the indices 1·74 and 1·99. The former must be that of the ordinary ray, and that of the extraordinary $1{\cdot}74 \times 1{\cdot}99 = 1{\cdot}86$. The great excess in both cases over Senarmont's results cannot, I think, be due to any error in my observations, since those made by different methods and at different times closely agree. Perhaps it is really due to some variation in the relative amounts of the bases in such a complex mineral. A comparison of these results with those obtained from the analogous section of calcite, shows very clearly the difference between positive and negative double refraction. They are as follows:—

		Double Refraction.		Ordinary.		Apparent extraordinary.
Calcite	...	Negative	...	1·659	...	1·335
Parisite	...	Positive	...	1·74	...	1·99

The apparent extraordinary index is thus abnormally small when the double refraction is negative, and abnormally large when positive.

Zircon cut obliquely to the axis shows the images of the circular hole widely separated. The ordinary ray is unifocal, and gives for the index about 1·975. The extraordinary ray is bifocal, and gives for lines parallel to the axis the index 2·083, and perpendicular to it 1·972. Neither of these can be the true index of the extraordinary ray, but since that for lines parallel to the axis is greater than that of the ordinary ray, they prove that this mineral has positive double refraction.

Quartz has such a weak double refraction that it is interesting chiefly as showing that it can be recognized by the method now under consideration. In a section $\frac{1}{5}$th of an inch thick, cut perpendicular to the axis, it is just possible to recognize two foci, and in one cut parallel to the axis to see that there is both a unifocal and a bifocal image. The true index of refraction of the extraordinary ray, deduced from lines parallel to the axis, exceeds by about 0·01 that of the ordinary ray, thus showing that this mineral has positive double refraction. My measurements indicate a well-marked difference in the actual value of the indices in different specimens, varying from about 1·55 to 1·56, that of the extraordinary ray being about 0·01 more in each case.

Proustite (ruby-silver-ore). A section parallel to the principal axis shows extremely well similar phenomena to those described in connexion with calcite. The index for the ordinary ray, polarized in the plane of the axis, is 2·98. The extraordinary ray, polarized in the opposite plane, gives for the true index 2·66, and for the apparent index for lines perpendicular to the axis 3·23. According to Professor Stokes's theory this should be $\frac{(2{\cdot}93)^2}{2{\cdot}66} = 3{\cdot}22$, which is as close an agreement as could be expected from indices determined by means of only a small crystal.

Tourmaline. A green, very dichroic specimen is interesting from the fact that it transmits only the extraordinary bifocal image, and thus appears to have no ordinary ray, which, however, is visible enough in less dichroic specimens.

Crystals with two Axes of Double Refraction.

Aragonite. On the whole I do not think I can select a better type than this mineral, because its double refracting power is so great, though the alternation of twin-crystals, shown by slight reflection at the planes of contact, and the intimate combination of similar twins, not visible by ordinary light, make it extremely difficult to determine the indices accurately. They also give rise to curious irregularities in the various phenomena now to be described. According to previous observers, the three true principal indices of refraction are 1·689, 1·684, and 1·531.

1. *Section cut perpendicular to the principal axis.*—My specimen is about $\frac{2}{5}$ths of an inch thick, sufficiently free from visible thin twin-plates, but yet by no means having a perfectly regular structure. On looking at the small circular hole, we at once see that the properties of the crystal are totally unlike anything so far described. As in the case of calcite cut perpendicular to the axis, there are two widely separated focal points, but instead of each showing a simple, well-defined circle, each shows a cross, due to the circles being elongated into two bands at right angles to one another. It is only in those parts of the specimen which are the most free from crystalline irregularities, that the cross is anything like as regular as shown in fig. 5. In most parts the arms are broken up into irregular branches, which appear to be due to the combination of two sets at slightly different azimuths, caused by the alternation of the twin-plates, which are invisible with ordinary light. In the case of calcite neither of the two images is polarized, and a Nicol's prism over the eye-piece produces no effect; whereas in the case of aragonite at the proper azimuths one pair of the opposite arms of the above-named cross or the other pair is extinguished, thus proving that both beams are polarized, one in the plane of the resultant axes, and the other perpendicular to it.

On arranging the nicol so that each image may be examined alone, both are seen to have the same bifocal characters as those of the extraordinary ray in a section of calcite cut parallel to the axis. There are two widely separated focal points about 0·1250 of the thickness of the specimen apart, at which the circular hole is drawn out into a band, which at one focus is parallel and in the other perpendicular to the plane of polarization.

On examining the grating we also find that the phenomena seen in each of the beams are the same as in the extraordinary ray of calcite. There are two focal points for the two systems of lines, but only at particular azimuths, and they disappear on being rotated to other azimuths. In the line of the axis there is thus no *ordinary ray*, and this explains what at first surprised me very much—why at certain azimuths it is impossible to see any trace of lines through a perfectly transparent substance, through which, with the naked eye, distinct objects are well seen in all positions. On careful examination it is found that the two extraordinary rays are very nearly equal, and differ chiefly in being in opposite planes. The result is that, when combined together, or when no nicol is used, both systems of lines are visible at so nearly the same focus, that they look very much like the two different unifocal images seen in a section of calcite cut perpendicular to the axis. That they are not so is, however, at once seen on rotating the lines, or on placing the nicol over the eye-piece.

The apparent indices of refraction were found to be approximately as follows:—

	Polarized in one plane.		Polarized in the opposite plane.
For one set of lines	1·676	...	1·388
For the other set of lines ...	1·384	...	1·677

According to Professor Stokes's theoretical deductions, assuming the three principal true indices to be as first given, these numbers should be respectively,

1·684	...	1·388
1·392	...	1·689

Considering the many sources of error, and that the specimen is certainly a mixture of crystals at different azimuths, the agreement is as close as could be expected, and at all events sufficient to show that the general principles of both theory and observation are correct.

2. *Section cut somewhat obliquely to the principal axis.*—This section shows the same general facts as that just described, but the two images

are much less symmetrical. The indices were found to be approximately as follows:—

	Polarized in one plane.		Polarized in the opposite plane.
For one set of lines	1·686	...	1·390
For the other set of lines ...	1·422	...	1·670

The chief point of interest in this case is that it proves that the actual value of some of the indices may be considerably different, if the direction of the section vary.

3. *Section cut parallel to the principal axis, and perpendicular to the plane of the twin-plates.*—The section was cut in this direction in order to avoid as much as possible the disturbing effect of the visible twins. The two rays, polarized in opposite planes, are both very decidedly bifocal, and the apparent indices were found to be approximately as follows:—

	Polarized in the plane of axis.		Polarized perpendicular to axis.
For lines parallel to axis ...	1·649	...	1·521
For lines perpendicular to axis ...	1·684	...	1·852

We here see very nearly the same abnormal increase of one of the indices, as in the case of calcite. Those having the values 1·521 and 1·684 closely approximate to two of the true indices.

4. *Section cut in the plane of one of the faces of the six-sided prism, parallel to the axis, but inclined at about* 60° *to the plane of the twinning.*—This section really gives the combined effects of crystals in somewhat different positions, though they are all combined together as twins in one particular plane. Its chief interest is that one image is so nearly unifocal that, without stopping off half the object-glass, by means of the cap previously described, it can scarcely be distinguished from an ordinary ray, with which it also agrees in being polarized in the plane of the axis. It is important to bear this fact in mind, since, if a crystal were inadequately examined only in such a direction, it might be thought to have only one axis of double refraction, when it really had two, as would be at once apparent if it were possible to examine the specimen in some other appropriate direction. These observed facts agree with what Professor Stokes has deduced theoretically. The apparent indices were found to be approximately as follows:—

	Polarized in the plane of axis.		Polarized perpendicular to axis.
For lines parallel to axis ...	1·664	...	1·521
For lines perpendicular to axis ...	1·664	...	1·795

One of these indices perhaps represents a real one. The rest are certainly only apparent.

It will thus be seen that the phenomena presented by crystals having two axes of double refraction are very complicated. Usually there are four different apparent indices; and though, according to the direction of the section, one or two of these may closely correspond with two of the three real indices, it would often require much care to distinguish the real from the apparent, especially if the true direction of the axes of the crystal were unknown. The existence of two axes of double refraction could, however, generally be proved by the two bifocal images.

Other Crystals having two Axes of Double Refraction.

Nitre. My sections, cut perpendicular to the axis, were made for a different purpose, and at the two focal points the lines at right angles to each other are only just sufficiently separated to prove that there are two axes of double refraction. I mention this merely as an example of what occurs in a biaxial crystal which differs but little from a uniaxial.

Barytes. This mineral is in some respects very suitable for examination, being easily procured, transparent, and free from irregularities. It can also be easily cut and polished. The only objection is that the difference in the indices is small. According to Heusser they are for the central red 1·645, 1·635, and 1·634. In the case of a section cut perpendicular to the principal cleavage, in a plane bisecting the acute angle formed by the two secondary planes of cleavage, which is in fact a section in the plane of two of the axes of the crystal, the apparent indices of refraction were found to be as follows:—

	Polarized parallel to the chief cleavage.	Polarized in the opposite plane.
For lines parallel to the cleavage ...	1·635 ...	1·648
For lines perpendicular to the cleavage	1·648 ...	1·630

In the case of a section cut in a plane perpendicular to the above, that is to say, in the plane of two of the axes of the crystal, perpendicular to the chief cleavage and bisecting the obtuse angle of the secondary cleavage, the indices were found to be:—

	Polarized parallel to the chief cleavage.	Polarized in the opposite plane.
For lines parallel to the cleavage ...	1·658 ...	1·633
For lines perpendicular to the cleavage	1·658 ...	1·682

In the case of a section cut parallel to the plane of the chief cleavage, the indices were found to be :—

	Polarized in the plane of the shorter diagonal of secondary cleavage.		Polarized in the plane of the shorter diagonal.
For lines parallel to the shorter diagonal	1·645	...	1·634
For lines parallel to the longer diagonal	1·645	...	1·656

In the case of a section cut in the plane of the secondary cleavage, the indices were :—

	Polarized in the plane of the chief cleavage.		Polarized in the opposite plane.
For lines parallel to the chief cleavage	1·636	...	1·644
For lines perpendicular to the chief cleavage	1·650	...	1·659

The equality in focal length of two sets of lines, as seen through the section first described, is not in any way an indication of a true or even an apparent unifocal image, since the equal foci were for pencils polarized in opposite planes. In only one out of the four different sections could there be any doubt about the existence of two bifocal images; and, even in that case, the unifocal character of one of the images was at once seen to be only apparent when the semicircular stop was used. Considering that two of the indices differ only by 0·001, it is satisfactory to find that the simple method of examination described in this paper gives such very distinct and positive results.

The apparent unifocal character of one image, in one instance, is, however, a further illustration of the necessity of a careful examination before concluding that any crystal, showing such an image, has only one axis of double refraction.

Mica (*muscovite*). A section in the plane of cleavage, which is perpendicular to the principal axis of the crystal, shows the circular hole as a symmetrical cross at two different foci. One branch is polarized in the line of the axis joining the centres of the coloured rings seen with polarized light, and the other perpendicular to it. This is the line of axis to which I refer below, in giving the indices. If the different branches of the crosses are at different foci, they differ so little that I was unable to recognize the difference, even in an excellent specimen $\frac{1}{5}$th of an inch thick.

	Polarized in the plane of this axis.		Polarized perpendicular to it.
For lines parallel to the above-named axis	1·528	...	1·598
For lines perpendicular to it ...	1·598	...	1·528

Orpiment. The double refraction is so powerful that a section in the plane of cleavage only $\frac{1}{20}$th of an inch in thickness shows four distinctly different foci. The circular hole is seen as an unsymmetrical cross at two chief foci, but unlike mica the two sets of arms are not in focus at the same time. The two images polarized in opposite planes are most distinctly bifocal, and give the following approximate indices, two of which are probably true, and the others only apparent:—

	Plane of polarization parallel to one axis.		Plane of polarization parallel to the other.
For lines parallel to one axis ...	2·09	...	3·03
For lines parallel to the other axis	2·68	...	1·81

Peridot. My section is cut parallel to one of the principal axes, but the specimen does not show sufficiently well the direction of the other axis. Two widely separated images of the circular hole are seen, both of which have the characters of extraordinary rays. The apparent indices were found to be:—

	Polarized in the plane of the axis.		Polarized in the opposite plane.
For lines parallel to the axis ...	1·654	...	1·679
For lines perpendicular to it ...	1·728	...	1·616

These all differ so much that there can be no doubt of the existence of two bifocal images, and that the mineral has two axes of double refraction.

Topaz. A section cut perpendicular to the principal axis shows the circular hole at two different levels, but neither is sensibly distorted. The apparent indices of refraction were found to be approximately as follows:—

	Polarized in the plane of the shorter axis.		Polarized in the opposite plane.
For lines parallel to shorter secondary axis	1·621	...	1·606
For lines parallel to the longer secondary axis	1·636	...	1·649

A section cut perpendicular to the principal axis, but doubtfully placed with reference to the other axes, gave the following indices:—

	Polarized in the plane of the principal axis.		Polarized in the opposite plane.
For lines parallel to the principal axis	1·618	...	1·635
For lines perpendicular to this axis	1·639	...	1·638

This case is interesting as showing two nearly equal indices for lines in the same direction, with the light polarized in opposite planes. Both this and the former do, however, clearly show that the method of study now being described enables us easily to prove that there is no true ordinary ray in topaz, although its double refraction is comparatively weak.

Anhydrite. A section cut parallel to the longest of the three axes and to one of the other axes, gave the following approximate indices:—

	Polarized in the plane of the chief axis.		Polarized in the opposite plane.
For lines parallel to the chief axis	1·587	...	1·548
For lines perpendicular to this axis	1·617	...	1·631

In the case of a section cut in the plane of the principal cleavage the apparent indices were:—

	Polarized in the plane of (1).		Polarized in the plane of (2).
For lines parallel to one of the shorter axes (1)	1·593	...	1·654
For lines parallel to the other (2) ...	1·691	...	1·601

The structure of this mineral makes it somewhat difficult to determine the indices with accuracy, and perhaps there ought not to be so many as the eight different values given above. There cannot, however, be any doubt as to the existence of two well-marked bifocal images, and of at all events six different indices, some probably true, and some only apparent.

Selenite. The double refraction of this mineral, when cut parallel to the principal cleavage, is so weak, that a specimen $\frac{3}{10}$ths of an inch thick shows one pair of lines only just at different foci, and the other pair only at one focus. The indices were found to be approximately as follows:—

	Polarized in the plane of (1).		Polarized in the plane of (2).
For lines parallel to one axis (1) ...	1·536	...	1·532
For lines parallel to the other (2) ...	1·527	...	1·532

With the semicircular stop it was just possible to distinctly recognize the bifocal character of even the second image. The chief interest of this mineral is in its showing that such small *relative* differences can be recognized. They correspond to $\frac{1}{1000}$th of an inch or less in focal length, and there can be no doubt of this, when it is seen in the self-same specimen.

Adularia. I mention this as an example in which the double refraction was so small that I was unable to prove the existence of any true bifocal image with a specimen 0·15 inch thick. It gave merely the one index 1·540. In reference to this fact I may here remark that the method of examination described in this paper shows the difference between the different systems mainly by differences in focal lengths, and, if these should in any case be considerably less than $\frac{1}{1000}$th of an inch, it may not be possible to recognize them. One result of this is that we may not be able easily to separate all minerals into perfectly separate groups, though the facts are no less characteristic of each particular kind.

Clinochlore. This mineral, cut perpendicular to the cleavage, may be given as an example of the effects of very strong dichroism, which is at once apparent when the circular hole is examined; but, if it were not taken into account, an observer might be misled. One image is not much coloured, but the other is so very dark that we may say that light is transmitted polarized only in one plane. Its laminar structure interferes much with the definition of lines seen through it, but some parts were sufficiently solid to give good results. The single image shows the two systems of lines at so nearly the same focus, that, without the semi-circular stop, no difference could be seen with a specimen $\frac{1}{8}$th of an inch in thickness. We thus get only one index of refraction, 1·595, as though the crystal had no double refraction. This is, however, manifestly because the other image is too dark to be visible.

None of the various other minerals which I have been able to examine show any facts differing in general character from those now described, and it therefore appears to me unnecessary to give further details. On the whole the phenomena are so clearly connected with double refraction that I think we may safely connect them with it in the following manner:—

General Relations between the Images and Double Refraction.

1. Crystals having no double refraction have no bifocal image.

2. Crystals having one axis of double refraction have one bifocal image.

3. Crystals having two axes of double refraction have two bifocal images.

4. Other circumstances being the same, the distance between the foci varies directly as the intensity of the double refraction, and directly as the thickness of the specimen.

Exceptions.—The only apparent exceptions to these laws hitherto met with are those dependent on such strong dichroism that only one image can be seen.

Though the inverse of all the above laws is probably true, yet it must be borne in mind that in some cases special means may be necessary in order to distinguish between an apparent and a true unifocal image. It must also be remembered that, since the distance between the foci in a bifocal image varies as the thickness of the mineral, it may not be possible to recognize it, if the specimen be too thin.

Application of the above described Methods and Conclusions to Practical Mineralogy.

The whole subject having grown up during the last two months, my attention has been so much directed to the establishment of general principles and methods that I have had no time to apply them to the determination of doubtful specimens. It is, however, quite apparent that even independent of accurate measurements, the methods described in this paper enable us to observe a new class of properties, which are most eminently characteristic of each particular mineral. Unlike some of the most important phenomena seen with polarized light, many of these properties can be observed in sections cut in any direction whatever, and in many cases the natural crystals need not be cut at all, comparatively small as well as larger specimens may be used, without previous preparation or permanent injury. This is of course a very great practical advantage. It is only necessary that there should be two opposite sufficiently flat faces, and that in some one part the crystal should be sufficiently transparent over a surface not necessarily more than $\frac{1}{20}$th of an inch square. Perfect freedom from impurities is by no means essential. Many good observations may be made with specimens loaded with fluid-cavities and minute crystals or granules. What is of far greater importance is that the mineral should not have a laminar or fibrous structure, giving rise to optically-discontinuous planes in any one predominating direction, much inclined to the plane of the section, since they produce distorted images of the lines by internal reflections. A very considerable amount of irregularity in the surface of the crystals may be overcome by the use of oil of cassia, or some other liquid of nearly the same refractive power as the mineral, placed between it and both the thick supporting glass and the thin cover. Results thus obtained might be only approximately true, but yet sufficiently to decide a doubtful question, when it was desirable not to polish the faces artificially. This is, however, in some cases very desirable or even necessary. Irregularities in the surfaces are objectionable in more ways than one, since they not only interfere with definition, but also with the correct measurement of the thickness of the specimen. I must, however, say that some of my best results have been obtained with crystals in their natural state; and, where their faces are good, they are far better than specimens badly cut and polished. One great advantage in natural crystals is that the planes are in a true crystalline direction, whereas, in cutting a specimen, much care is necessary to avoid errors due to a departure from the true plane. The opposite surfaces ought to be flat and parallel, but a slight inclination does not produce any appreciable error. On the whole it is far better to mount the specimen on glass, and cover it with a thin glass, cementing with Canada-balsam, since this greatly reduces the effects of irregularities.

Many very valuable facts may, however, be learned from a specimen cut in an unknown direction. It may show, not only whether the crystal has or has not double refraction, but also whether it has one or two axes. When there is no double refraction, or only one axis, the index of refraction of the ordinary ray can always be determined, and may assist in the identification of the mineral; or if there be no doubt about this, a difference in the index may point out some important variation in the chemical composition. By using the round hole and the grating the direction of the section may be approximately determined, and also the general character of the double refraction, whether strong or weak, positive or negative. In the case of crystals having two axes of double refraction, the results are necessarily more complicated; but, even when the direction of the section is unknown, many important facts may be observed. If, however, we can examine a natural or artificial specimen from $\frac{1}{10}$th to $\frac{1}{2}$ inch thick, in one or more known directions, the phenomena I have described are so very definite and characteristic that they appear to me likely to be of very considerable use in the identification of minerals.

The facts described in this paper also have an important bearing on the study of the microscopical structure of rocks. Only an ordinary ray, or an extraordinary ray differing so little from an ordinary as to be almost unifocal, could give perfect definition of cavities or enclosed crystals, seen through a relatively considerable thickness of any mineral having a powerful double refraction. It is therefore necessary not only to get rid of one of the two images by using polarized light, but so to arrange the plane of polarization as to make use of the ordinary ray, or of that extraordinary ray which is the most nearly unifocal. The importance of this does not depend merely on the absolute thickness of the mineral, but also on the magnifying power employed, since the distortion of minute objects, seen with high powers through a small thickness, would be as great as that of larger objects seen with a lower power through a greater thickness, and might give rise to very false appearances. Though in most minerals this source of error may be disregarded, yet at the same time it must be borne in mind that cases might occur in which it would be impossible to see a small object, well defined, by any means hitherto adopted.

Fig. 1. 2. 3. 4 *a*. 4 *b*.

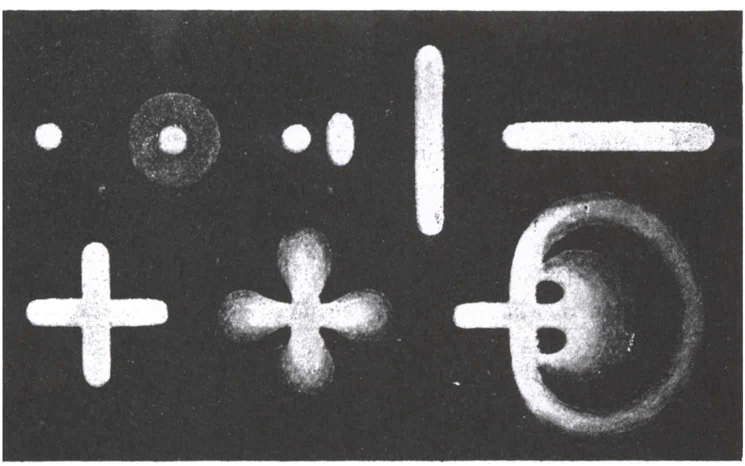

Fig. 5. 6. 7.

Description of the Figures.

The subjoined explanations will be better understood after reading the more full account of the various special examples, but it appears desirable to give some description of the different figures.

Fig. 1 is the simple hole as seen through a mineral having no double refraction (p. 196).

Fig. 2 shows one of the images seen in a section of calcite cut perpendicular to the axis, surrounded with the larger nebulous circle, due to the other image being much out of focus (p. 197).

Fig. 3 is what is seen where a section of one uniaxial crystal is cut obliquely to the axis. The ordinary image is circular, and remains in the centre, whilst the extraordinary is somewhat distorted, and, by inversion, as it were, raised in the line of the axis. The fig. represents the effect of a thin slice of a mineral in which the double refraction is powerful, or of a thick slice of one in which it is weak (p. 199).

Fig. 4 shows at *a* and *b* the two very much elongated images of the circle, seen by means of the extraordinary ray, through a section of calcite cut parallel to the axis. These two images are at two different, widely-separated foci. If both could be seen at the same time, they would form a cross similar to fig. 5. (p. 200).

Fig. 5 represents one of the two crosses, seen at two widely-separated foci, in looking through a section of such biaxial crystals as aragonite or mica, cut perpendicular to the vertical axis. The circular hole is drawn out in two directions, at right angles to each other, parallel to the axes of the crystal, into two beams polarized in opposite planes (p. 204). The irregularities in the structure of aragonite often cause the arms of these crosses to end in irregular brushes.

Fig. 6.[1] Section of aragonite cut perpendicular to the vertical axis. The spreading out of the ends of the arms of the cross is here apparently due to the crystal consisting of portions having their axes nearly but not absolutely parallel.

Fig. 7.[1] Section of aragonite cut somewhat obliquely to the vertical axis. Here one bar of the cross is distorted into an irregular arc, and one arm of the other bar is spread out into a sort of crescent.

[1] Dr. Sorby's MS. contains no explanation of figs. 6 and 7; those now given being taken from Min. Mag., 1877, vol. i, p. 200. All the above figures were first given in Plate VII of that volume, and they are reproduced here for convenience of reference.