# *The energy of twin-crystals.*

By HAROLD HILTON, M.A.

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VERNADSKY[1] has pointed out that in certain cases a twin-crystal appears to be a more stable form than the simple polyhedron, as proved by Scacchi's observation that twins sometimes exceed in magnitude and regularity simple crystals growing in the same solution. He suggests that an explanation can be found in the fact that the surface-energy may be less for a twin than for a simple crystal of the same volume. The suggestion is a valuable one, but Vernadsky applies no quantitative test to his theory. I have therefore thought it worth while to work out the simplest case, which seems to be the following :—

Suppose we have a simple crystal as shown in fig. 1, and also a twin by reflection on a face $f$ (the right-hand face of fig. 1) having the same

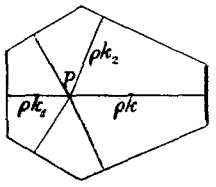


Fig. 1.

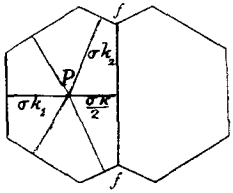


Fig. 2.

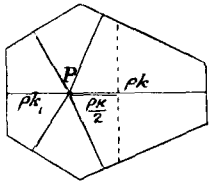


Fig. 3.

combination of faces as the simple polyhedron except that $f$ is now a composition-plane. Then for the simple crystal it is known that, if $k$, $k_1$, $k_2$, $k_3$, ... are the capillarity-constants (surface-energies per unit area) of the faces, there is a point $P$ inside the crystal whose distances from the faces are $\rho k$, $\rho k_1$, $\rho k_2$, $\rho k_3$, ... ; where $\rho = 3V/E$, $V$ being the volume of the crystal and $E$ its total surface-energy.[2] This is assuming that disturbing influences are eliminated during growth, so that the crystal may be considered as having its normal habit, in which

[1] W. Vernadsky, 'Beiträge zur Energetik der Krystalle.' Zeits. Kryst. Min., 1908, vol. xlv, pp. 124–142.

[2] See Hilton's 'Mathematical Crystallography', 1903, p. 107. With the notation there used $n'_t = \rho k_t$, and therefore $3V = \Sigma n's = \rho \Sigma ks = \rho E$.

the total surface-energy for a given volume and combination of faces is a minimum.

To find the normal shape of the twin-crystal of fig. 2, let $\kappa$ be the surface-energy per unit area of the composition-plane. Then we may assume that the shape is the same as if, firstly, each of the component crystals had grown independently so that the face $f$ had the capillarity-constant $\kappa/2$, and the remaining faces had capillarity-constants $k_1, k_2, k_3, \ldots$ as before; and, secondly, the crystals were afterwards united along the face $f$ without gain or loss of energy. Then there is a point inside each crystal of the twin such that the perpendiculars from it on the composition-plane and faces are $\sigma\kappa/2$, $\sigma k_1$, $\sigma k_2$, $\sigma k_3$, ..., where

$$\sigma = \frac{3 \times \text{volume of each crystal}}{\text{surface-energy of each crystal}} = \frac{3 \times \text{volume of whole twin}}{\text{surface-energy of whole twin}}.$$

If the simple polyhedron and twin have equal volumes, the former or the latter has the less surface-energy, i. e. is the more stable, according as $\rho$ or $\sigma$ is the greater.

In other words (remembering that the ratio of volume to energy increases as the volume increases[1]), if in fig. 1 the plane parallel to $f$ and distant $\rho\kappa/2$ from $P$ cuts off more than half the crystal, the twin is more stable than the simple crystal with the same volume (see fig. 3).

If the crystal of fig. 1 has a symmetry-plane parallel to $f$, the twin by reflection becomes a parallel growth. In this case $\kappa = 0$, and $P$ is the centre of the crystal. We see that two crystals forming a parallel growth will be in general less stable than the single crystal of the same volume.

The following is a list of some papers on this subject. Quantitative experiments on the habit of crystals are badly needed, and would doubtless lead to valuable results.

P. CURIE, Bull. Soc. franç. Min., 1885, vol. viii, p. 145.

G. WULFF, Zeits. Kryst. Min., 1901, vol. xxxiv, p. 520.

H. HILTON, Centralblatt Min., 1901, p. 753; Phil. Mag., 1902, ser. 6, vol. iii, p. 144.

W. VERNADSKY, Zeits. Kryst. Min., 1908, vol. xlv, p. 124.

Bedford College,
London.

[1] The volume increases as the cube, and the surface-energy as the square of the linear dimensions of the crystal.