

Axes of symmetry and the crystallographic classes.

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It is usual to divide axes of rotatory symmetry into those of the first and those of the second sort. In the former a crystal is brought into coincidence by a rotation through a certain angle round the axis and in the latter by such a rotation followed by a reflection about (or, as I prefer to say, a reversal relatively to) the plane at right angles to the axis. In 1906 Mr. H. Hilton¹ proposed to define an axis of rotatory symmetry of the second sort as one in which the rotation was followed by an inversion about (reversal relatively to) a point, and he showed that his proposal would, if adopted, simplify the definition of the crystallographic classes.

The reason for this change is most easily understood if instead of distinguishing two kinds of axes we discriminate between two degrees of coincidence. For this purpose the crystallographic structure is considered as expressed by lines; of which the more important are obviously those parallel to possible crystal-edges or normal to possible faces.

In complete or *co-directional* coincidence not only are equivalent lines brought into coincidence but also equivalent directions in those lines. Whereas in incomplete or *contra-directional* coincidence equivalent lines coincide but the equivalent directions of uniterminal lines are brought into opposition to one another. Both kinds of coincidence may be described as co-linear coincidence, for in both equivalent lines coincide.

If a crystal can be brought into co-directional coincidence by a rotation through an angle $2\pi/n$ and no smaller angle, the axis will be in the language employed by Mr. Hilton an *n*-al rotation axis of the first sort, or, as I should prefer to call it, an axis of co-directional rotatory symmetry with cyclic number *n*.

If, on the other hand, the coincidence be only contra-directional, the axis will be in Mr. Hilton's words an *n*-al axis of the second sort, or, from my point of view, an axis of contra-directional rotatory symmetry

¹ H. Hilton, *Mineralogical Magazine*, 1907, vol. xiv, p. 261.

with cyclic number n . In both cases it may also be described as an axis of co-linear rotatory symmetry with the same cyclic number n .

It is easy to show that an axis of contra-directional rotatory symmetry with cyclic number n is an axis of co-directional rotatory symmetry with cyclic number $n/2$.

The use of these terms may be illustrated by the diagrams of crystals referred to in the following paper (p. 401) on 'A modification of the stereographic projection'.

In the holosymmetric class of the cubic system (1a and 1b) the crystallographic axes are axes of co-directional and co-linear rotatory symmetry with cyclic number 4. In the tetrahedrite class of the same system (2a and 2b) they are axes of contra-directional and co-linear rotatory symmetry with cyclic number 4 and axes of co-directional rotatory symmetry with cyclic number 2.

In the calcite class of the rhombohedral system the principal axis is an axis of co-directional and co-linear rotatory symmetry with cyclic number 3, whereas in the benitoite class of the same system it is an axis of contra-directional and co-linear rotatory symmetry with cyclic number 6 and one of co-directional rotatory symmetry with cyclic number 3.

In a paper on 'A simple arrangement and notation of the thirty-two classes of symmetry based on the symmetry of zone-axes'¹ I have given a table of crystallographic classes arranged according to the co-directional cyclic numbers of the axes of rotatory symmetry with the highest cyclic number, except in the case of the cubic symmetry where the four axes with cyclic number 3 are employed. The accompanying table is exactly similar except that it is based on the co-linear cyclic number, and the letter *k* indicates that the axis is one of contra-directional rotatory symmetry. Not only is the arrangement simpler, there being a smaller number of rows, but it has the advantage that all the members of a system appear in the same column: the first column comprising the triclinic, the second the monoclinic and orthorhombic, the third the tetragonal, and so on. The trigonal bipyramidal and ditrigonal bipyramidal (benitoite) classes find a place with the hexagonal instead of the rhombohedral (trigonal) classes. Groth's nomenclature is appended in each case.

Similar symbols may be employed to designate different types of rotatory axes, but for the sake of distinction Arabic instead of Roman numerals should be employed.

¹ J. W. Evans, *Mineralogical Magazine*, 1907, vol. xiv, pp. 360-364.

Co-linear cyclic number:	System.	Simple or one whole turn axis.	Digonal or one-half turn axis.	Tetragonal or one-quarter turn axis.	Trigonal or one-third turn axis.	Hexagonal or one-sixth turn axis.	Four trigonal or one-third turn axes.
		I.	II.	IV.	III.	VI.	C + (4 × III)
		Triclinic.	Monoclinic and Orthorhombic.	Tetragonal.	Rhombohedral.	Hexagonal.	Cubic.
Unilateral (U). Uniterminal (u).		I Uu Asymmetric. <i>Calcium thio-sulphate.</i>	II Uu Sphenoidal. <i>Cane-sugar.</i>	IV Uu Tetragonal pyramidal. <i>Wulfenite.</i>	III Uu Trigonal pyramidal. <i>Sodium periodate.</i>	VI Uu Hexagonal pyramidal. <i>Nepheline.</i>	C Uu Tetrahedral-pentagonal-dodecahedral. <i>Ulmamine.</i>
Biterminal. Central (c).		I Uc Pinacoidal. <i>Abité.</i>	II Uc Prismatic. <i>Aspidé.</i>	IV Uc Tetragonal bipyramidal. <i>Scheelite.</i>	III Uc Rhombohedral. <i>Phenacite.</i>	VI Uc Hexagonal bipyramidal. <i>Apatite.</i>	C Uc Dyakis-dodecahedral. <i>Iron-pyrites.</i>
Contra-directional (k).		—	II Uk Domatic. <i>Clinochroite.</i>	IV Uk Tetragonal bisphenoidal. <i>No example known.</i>	—	VI Uk Trigonal bipyramidal. <i>No example known.</i>	—
Helical (h).		(See II Uu)	II Uh Bisphenoidal. <i>Epsomite.</i>	IV Uh Tetragonal trapezohedral. <i>Strychnine sul/hate.</i>	III Uh Trigonal trapezohedral. <i>α-quartz.¹</i>	VI Uh Hexagonal trapezohedral. <i>β-quartz.¹</i>	C Uh Pentagonal-icositetrahedral. <i>Cuprite.</i>
Bilateral (B). Uniterminal (u).		(See II Uu)	II Bu Pyramidal. <i>Hemimorphite.</i>	IV Bu Ditetragonal pyramidal. <i>Six-cin-toctimide.</i>	III Bu Ditrigonal pyramidal. <i>Pyroxenite.</i>	VI Bu Dihexagonal pyramidal. <i>Iodrite.</i>	C Bu Hexakis-tetrahedral. <i>Tetraedrite.</i>
Biterminal. Central (c).		(See II Uc)	II Bc Bipyramidal. <i>Staurolite.</i>	IV Bc Ditetragonal bipyramidal. <i>Rutile.</i>	III Bc Ditrigonal scalenohedral. <i>Calcite.</i>	VI Bc Dihexagonal bipyramidal. <i>Beryl.</i>	C Bc Hexakis-octahedral. <i>Spinel.</i>
Contra-directional (k).		—	(See II Bu)	IV Bk Tetragonal scalenohedral. <i>Chalcopyrite.</i>	—	VI Bk Ditrigonal bipyramidal. <i>Benitoite.</i>	—

¹ α-quartz stable below 570°C., β-quartz stable above 570°C. (Cf. O. Mügge, Neues Jahrb. Min., 1907, Festband, p. 181.)