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The dispersion phenomena and the influence of temperature on the optic axial angle of Sanidine from the Eifel.

(*With Plates IX and X.*)

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IN the course of my study of the dispersion phenomena of the felspar group in the Mineralogical Laboratory of the University of Cambridge, Professor Lewis advised me to study the remarkable properties of sanidine from the Eifel, which were first noticed by Des Cloizeaux, and he very kindly placed the mineral sections in the Museum at my disposal. In order to be certain of the material used, application was made to Professor A. Lacroix of Paris, who with the courtesy and generosity characteristic of his countrymen was good enough to send some fragments of the very crystals used by Des Cloizeaux in his classical researches. I tender him my most hearty thanks. My hearty thanks are also due to Mr. A. Hutchinson for his constant help.

Dispersion of the Refractive Indices.

The refractive indices of the Eifel sanidine, for a definite wave-length or for different portions of the spectrum, have been already studied by several authors, either by the method of total reflection or by that of minimum deviation. As the principal indices are liable to differ with the specimen, those of No. 1, a specimen belonging to the Cambridge Museum, were determined in the following way: The refractive indices α and γ for six different wave-lengths were found by the Abbé-Pulfrich total-reflectometer—the instrument used in the determination of the refractive indices of the Rischuna albite.[1] The mean index β could not be thus determined, for the shadows due to the β and γ beams fall too closely together, consequently this index has been computed from the observed values of α and γ combined with the measured angle $2E$.

The data of the crystal No. 1, in Table I, were obtained at a temperature of nearly 19° C. with a section practically perpendicular to the acute bisectrix for $\lambda = 508{\cdot}5\ \mu\mu$. As at this temperature the axial planes were perpendicular to the plane of symmetry for all parts of the spectrum used, the adjustment of the plate was made once for all, and all that was needed was to change the colour of the light.

Table I.[2]

Observed α and γ, and deduced β.

(Temperature 19° C., nearly.)

λ in $\mu\mu$	α	β	γ
486	1·5265	1·5312	1·5312
508·5	1·5249	1·5298	1·5298
535	1·5234	1·5280	1·5280
589 (Na)	1·5206	1·5251	1·5252
644	1·5189	1·5232	1·5233
671	1·5180	1·5225	1·5226

[1] S. Kôzu, 'The dispersion phenomena of albite from Alp Rischuna, Switzerland.' Mineralogical Magazine, 1915, vol. xvii, pp. 189–192.

[2] For comparison are given below the refractive indices of the sanidine from Wehr, Eifel, determined by A. Mülheims (Zeits. Kryst. Min., 1888, vol. xiv, p. 235):—

		α		β and γ
F (486)	...	1·52556	...	1·53010
b (518)	...	1·52354	...	1·52813
D (589)	...	1·51984	...	1·52439
C (656)	...	1·51746	...	1·52202
B (687)	...	1·51667	...	1·52100

Dispersion of the Optic Axial Angle and the Optic Axial Plane.

The optic axial angles for different wave-lengths were measured in the two specimens No. 1 and No. 2 by Hutchinson's universal goniometer.[1] Section No. 2 was prepared from one of the Paris specimens examined by Des Cloizeaux, and was about $13 \times 7 \times 3$ mm. It differed from No. 1 inasmuch as at 19° C. the axial plane for long wave-lengths was perpendicular to (010), whilst for short ones it was parallel to (010). Furthermore, the axial angle varied in the different parts of the section for one and the same colour, and this change seemed to take place gradually.

Table II shows the dispersion of the optic axial angle of No. 1 at 19·4° C., and those of four different portions, *b*, *c*, *d*, *e* of section No. 2, at temperatures ranging between 19·4° C. and 19·8° C.

Table II.

E observed in the two specimens No. 1 *and No.* 2.

λ in μμ	No. 1 (*t.* = 19·4° C.)	No. 2 (*t.* = 19·4°–19·8° C.)			
	Axial plane ⊥ (010)				A. P. ∥ (010).
	a	*b*	*c*	*d*	*e*
478	0° 0′	—	—	—	—
486	2 14·7	—	—	—	8° 28·3′
508·5	5 12·1	—	—	—	7 4·0
535	7 2·5	—	—	—	5 26·3
542	—	0° 0′	—	—	—
554	7 59·5	2 49·7	—	—	4 4·8
585	—	—	—	—	0 0
589	9 18·7	5 16·2	—	—	—
600	—	—	0° 0′	—	—
610	—	6 11·0	2 2·0	—	—
615	—	—	—	0° 0′	—
644	10 44·8	7 22·3	4 25·2	3 40·0	—
671	11 16·6	8 20·0	5 40·0	5 0	—

The dispersion of the axial plane in the specimen No. 1 was measured by an approximate method with the universal goniometer. The two optic axes for each wave-length were carefully adjusted so as to coincide with the vertical line of the eye-piece, while at the same time the axial angle was bisected by the horizontal wire, the distances of the axes being

[1] A. Hutchinson, 'A universal goniometer.' Mineralogical Magazine, 1911, vol. xvi, pp. 100–108.

determined by a micrometer scale. The deviations of the planes for different wave-lengths were then easily found by rotating the plate. As stated above, the plate was practically perpendicular to the acute bisectrix for $\lambda = 508{\cdot}5\ \mu\mu$, and the axial plane for this wave-length was, in the measurements, taken as standard for the deviations of the other planes. The values measured in air are given in Table III. The correction needed to give the real deviation in the crystal is with sufficient accuracy found by the equation:

$$\sin d_{,} = \frac{\sin d}{\beta},$$

where d = the observed deviation, $d_{,}$ = the real one, and β = the mean refractive index.

Table III.

Dispersion of the Optic Axial Plane.

(Temperature 18·8° C. – 19° C.)

λ in $\mu\mu$	Observed deviation (d)	Deduced deviation ($d_{,}$)
508·5	0′	0′
535	–15	–9·8
589	–38	–24·9
644	–1° 2	–40·7
671	–1 11	–46·6

The negative sign in the table denotes that on the side pinakoid (010) the angle is measured counter-clockwise from the standard bisectrix. The values of $d_{,}$ are also those of the dispersion of the elasticity-axes[1] in the symmetrical plane.

The Influence of Temperature on the Optic Axial Angle.

Des Cloizeaux noticed that at ordinary temperature the plane of the optic axes is in most of the crystals perpendicular to that of symmetry, and that the axial angle is small and rapidly diminishes to zero as the temperature is raised. Further, that the axes for blue light closed in first and then at higher temperatures for each colour in turn. As the temperature rises still further the optic axes open out in the symmetry-

[1] Readers interested in the problem of the dispersion of the elasticity-axes will find a theoretical explanation in S. Nakamura, 'Über die Dispersion der optischen Symmetrieachse im durchsichtigen inaktiven monoklinischen Krystall.' Physik. Zeits., 1905, vol. vi, pp. 172–174.

plane, the angle for red being always less than that for blue. He continued his observations up to a temperature of 342·5° C., and has given the corresponding angle for red light for numerous steps. He found that so far the changes were gradual and transitional, and that after a time the crystal returned on cooling to its original condition. He also found that if the plate were raised to the temperature of a porcelain furnace—about 600° C.—the axial plane remained permanently in that of symmetry and that on cooling the angle diminished only to some extent.

In my investigation of these phenomena I have used monochromatic light of seven definite wave-lengths, and the range of temperature has varied from 5° C. to 71° C., the temperature being thus narrowly restricted by the instrumental conditions. The plate was inserted in a cork which exactly fitted in the heating cell and the whole moved together as a plate. The cell was that used by Mr. Hutchinson and Dr. Tutton[1] in the investigation of the changes in gypsum, and was fixed to the rotatory axis in the field of Hutchinson's universal gonio-

Table IV.

486 μμ		5μμ		535 μμ		554 μμ		589 μμ		644 μμ		671 μμ	
t	*E*	*t*	*E*	*t*	*E*	*t*	*E*	*t*	*E*	*t*	*E*	*t*	*E*
5·9°	7°33′	5·4°	8°48′	5·7°	9°45′	5·4°	10°32′	5·7°	11°22′	5·3°	12°29′	5·3°	13°16′
14·8	4 54	14·8	6 33	14·8	7 53	14·8	8 39	10·0	10 30	14·5	11 6	17·4	11 9
17·3	4 4	17·3	5 56	17·2	7 21	17·1	8 12	14·2	9 53	17·4	10 45	19·7	10 49
19·3	3 30	23·3	4 10	23·0	5 58	23·1	6 51	15·3	9 51	19·9	10 13	25·2	10 8
22·5	0 0	26·1	2 22	26·7	4 41	26·5	5 57	17·0	9 24	25·5	9 26	33·0	8 43
25·4	3 10	27·5	0 0	30·6	2 55	30·3	4 43	19·9	8 55	32·8	7 53	38·3	7 29
30·5	5 18	31·3	3 45	33·0	0 0	34·1	3 8	25·0	7 48	38·1	6 38	41·9	6 29
31·7	5 46	34·8	4 57	35·3	2 49	37·0	0 0	26·2	7 38	41·7	5 35	47·4	5 1
34·6	6 40	40·5	6 34	39·8	4 44	39·0	2 13	32·3	5 50	44·2	4 42	51·6	3 4
41·1	8 12	45·5	7 49	45·5	6 25	45·2	5 11	35·5	4 40	47·3	3 34	53·8	0 0
45·6	9 15	52·5	9 22	52·3	8 6	51·3	7 3	39·5	3 24	51·7	0 0	57·5	3 53
52·0	10 25	55·3	9 43	57·6	8 59	57·6	8 14	42·0	2 20	53·2	2 42	60·4	4 27
				63·8	10 8	63·4	9 19	43·5	0 0	55·4	4 7	63·7	5 40
						68·7	10 13	45·3	2 38	57·4	4 32	66·2	6 30
								47·5	3 34	60·3	5 30	71·5	7 38
								51·5	5 20	63·5	6 30		
								55·0	6 13	66·5	7 15		
								60·5	7 28	71·5	8 47		
								66·6	8 55				
								71·6	9 48				

[1] A. Hutchinson and A. E. H. Tutton, 'On the temperature of optical uniaxiality in gypsum.' Mineralogical Magazine, 1912, vol. xvi, pp. 257–263.

meter. The observed angle is $2E$. In the experiment the sides of the cell and the crystal plate should all be parallel to one another, and also to the axis of the graduated circle. The adjustments are difficult and cannot be made with much accuracy, and the faults arising from this cause are the main source of the errors in the observations. For the temperatures and wave-lengths stated the observed values of E are given in Table IV (p. 241).

The variation with the temperature of E for each wave-length is graphically represented in Plate IX, which is a reduction of a diagram plotted on millimeter paper. In the diagram 5 mm. along the abscissae represent 1° C., and 10 mm. along the vertical ordinate an angle of 1°. The single circlets give actual determinations when the plate lies in the cell, the double circlets give those in which the plate alone was used. The black dots are points on the curves which correspond to temperatures differing from that of the zero-point of the curve by 10° C. and 15° C. The continuous curve (with λ constant) may be denoted as the $(E\ t)$ curve. Each of them has one point of inflexion, which corresponds to the zero position of the axial angle. On the opposite sides of this point the optic axial planes are perpendicular to one another, the plane at lower temperatures being at right angles to that of crystallographic symmetry, and at higher temperatures parallel to it. In the former position of the plane, the angle diminishes as the temperature rises, while in the latter the angle increases with the temperature.

Assuming the refractive indices to be linear functions of the temperature, Pockels[1] has shown that for a *rhombic* crystal the curve connecting temperature and axial angle (when the latter is small) is a parabola having its vertex at the zero-point of E. In the Eifel sanidine the displacement of the acute bisectrix for each colour at different temperatures was tested for the case where $\lambda = 589\ \mu\mu$. It was found to be so very small as to be negligible within the range of temperature employed, and the same has been assumed in every other case. The symmetry-axis of the crystal is for every colour necessarily a fixed principal axis. Hence as long as one colour only is considered, the portion of the $(E\ t)$ curve on each side of the uniaxiality point should also be a parabola. Within the limits of error in the observations they prove to be so; and each portion can be readily shown to lie between the two parabolas:— $E_{\prime}^2 = -3{\cdot}3\ t$, and $E_{\prime\prime}^2 = -3{\cdot}6\ t$; the negative sign being introduced to correspond with the diagram. These two parabolas are drawn in the

[1] F. Pockels, 'Lehrbuch der Kristalloptik,' 1906, p. 454.

figure, the zero-points having been placed arbitrarily at 65° C. and 70° C. The values of $E_{,}$ and $E_{,,}$ for different decrements of temperature are given in Table V, and for the sake of comparison their values have been read from the two parabolas at temperatures corresponding with those at which measurements of E were made. The three sets of values are given in Table VI (pp. 244–5). The accuracy attainable in the readings from the diagram is within the limits $\pm 6'$. With few exceptions (marked by stars) the observed values of E fall between the corresponding values of $E_{,}$ and $E_{,,}$. Hence in the several curves extremities of the ordinates corresponding to a definite increment or decrement of temperature should lie very approximately on a straight line parallel to the line of abscissae. That this is the case may be seen from the black dots which give ordinates for temperatures differing from those of the zero-points by 10° C. and 15° C. This may be put in another way; viz. that all the curves are, within the limits of accuracy attainable, identical, and can by a mere translation along the line of abscissae be superposed the one on the other.

To test the phenomena further, Plate X, fig. 1 has been drawn, in which the abscissae are wave-lengths, the ordinates E, and the temperature corresponding to each of the curves is that indicated at its extremity. The curves may be distinguished from the former ones by calling them the $(E\lambda)$ curves. They have been obtained from Plate IX, by plotting off the value of E corresponding to definite values of λ and t. As in Plate IX, the double circlets give actual observations on specimens Nos. 1 and 2, when the plate alone was used. The crosses correspond

Table V.

t	$E_{,}$	$E_{,,}$
1°	1·82°	1·90°
2	2·57	2·68
5	4·06	4·24
8	5·10	5·40
10	5·75	6·00
15	7·04	7·35
20	8·03	8·48
25	9·08	9·49
30	9·95	10·39
35	10·75	11·23
40	11·49	12·00
45	12·19	12·73

Table VI.

λ = 486 μμ				λ = 508·5 μμ			
t	$E_{,}$	E	$E_{,,}$	t	$E_{,}$	E	$E_{,,}$
5·9°	7°24′	7°33′	7°45′	5·4°	8°33′	8°48′	8°57′
14·8	4 54	4 54	5 18	14·8	6 27	6 33	6 45
17·3	4 6	4 4*	4 21	17·3	5 48	5 56	6 6
19·3	3 15	3 30*	3 24	23·3	3 39	4 10*	3 54
22·5	0 0	0 0	0 0	26·1	2 10	2 22*	2 20
25·4	3 6	3 10	3 12	27·5	0 0	0 0	0 0
30·5	5 6	5 18	5 24	31·3	3 30	3 45*	3 42
31·7	5 30	5 46	5 48	34·8	4 52	4 57	5 9
34·6	6 21	6 40	6 40	40·5	6 30	6 34	6 51
41·1	7 48	8 12	8 12	45·5	7 39	7 49	8 3
45·6	8 42	9 15*	9 9	52·5	9 18	9 22	9 42
52·0	9 51	10 25*	10 21	55·3	9 36	9 43	10 0

λ = 535 μμ				λ = 554 μμ			
t	$E_{,}$	E	$E_{,,}$	t	$E_{,}$	E	$E_{,,}$
5·7°	9°30′	9°45′	9°57′	5·4°	10°12′	10°32′	10°42′
14·8	7 42	7 53	8 6	14·8	8 30	8 39	8 57
17·2	7 24	7 21*	7 48	17·1	8 2	8 12	8 30
23·0	5 45	5 58	6 0	23·1	6 48	6 51	7 3
26·7	4 33	4 41	4 48	26·5	5 54	5 57	6 9
30·6	2 45	2 55	2 57	30·3	4 42	4 43	4 57
33·0	0 0	0 0	0 0	34·1	3 6	3 8	3 15
35·0	2 34	2 49*	2 41	37·0	0 0	0 0	0 0
39·8	4 42	4 44	4 57	39·0	2 34	2	2 41
45·5	6 24	6 25	6 42	45·0	5 10	5 11	5 27
52·3	7 54	8 6	8 21	51·3	6 54	7 3	7 9
57·6	9 0	8 59*	9 27	57·6	8 12	8 14	8 39
63·8	10 6	10 8	10 33	63·4	9 20	9 19	9 45
				68·7	10 12	10 13	10 45

Table VI (cont.).

λ = 589 μμ				λ = 644 μμ			
t	$E_{,}$	E	$E_{,,}$	t	$E_{,}$	E	$E_{,,}$
5·7°	11°12′	11°22′	11°39′	5·3°	12°27′	12°29′	12°57′
10·0	10 30	10 30	11 0	14·5	11 6	11 6	11 36
15·3	9 39	9 51	10 6	17·4	10 39	10 45	11 9
17·0	9 21	9 24	9 48	19·9	10 12	10 13	10 42
19·9	8 51	8 55	9 15	25·5	9 15	9 26	9 45
25·0	7 45	7 48	8 12	32·8	7 50	7 53	8 15
26·2	7 33	7 38	7 54	38·1	6 45	6 38*	7 0
32·3	6 3	5 50*	6 21	41·7	5 45	5 35*	6 0
35·5	5 6	4 40*	5 24	44·2	4 57	4 42*	5 12
39·5	3 33	3 24*	3 48	47·3	3 42	3 34	4 0
42·0	2 15	2 30*	2 20	51·7	0 0	0 0	0 0
43·5	0 0	0 0	0 0	53·2	2 15	2 42*	2 20
45·3	2 24	2 38*	2 33	55·4	3 30	4 7*	3 39
47·5	3 33	3 34	3 48	57·4	4 18	4 32	4 33
51·5	5 6	5 20	5 24	60·3	5 18	5 30	5 36
55·0	6 9	6 13	6 27	63·5	6 15	6 30	6 33
60·5	7 27	7 28	7 48	66·5	7 0	7 15	7 18
66·6	8 42	8 55	9 9	71·5	8 0	8 47*	8 24
71·6	9 36	9 49	10 3				

λ = 671 μμ			
t	$E_{,}$	E	$E_{,,}$
5·3°	12°45′	13°16′	13°18′
17·4	10 48	11 9	11 30
19·7	10 36	10 49	11 6
25·2	9 42	10 8	10 12
33·0	8 15	8 43	8 42
38·3	7 6	7 29	7 30
41·9	6 15	6 29	6 33
47·4	4 36	5 1*	4 48
51·6	2 39	3 4*	2 48
53·8	0 0	0 0	0 0
57·5	3 30	3 53*	3 39
60·4	4 39	4 27*	4 54
63·7	5 42	5 40*	5 57
66·2	6 23	6 30	6 42
71·5	7 35	7 38	7 57

to equal differences of abscissae measured from the zero-points, and lie very nearly on straight lines, which, if the curves of Plate IX were strictly equal parabolas, should be parallel to the line of abscissae. They are, however, very appreciably inclined to it and to one another.

Another curve, U, in Plate IX, shows the relation of uniaxiality of the crystal, the abscissae giving the temperature, and the ordinates the wave-lengths. Its empirical formula is:

$$t = 0{\cdot}2390\ \lambda - 0{\cdot}0003\ \lambda^2,$$

the origin of co-ordinates being at $t = 22{\cdot}5°$ C. and $\lambda = 486\,\mu\mu$.

THE DISPERSION FORMULA FOR THE OPTIC AXIAL ANGLE.

To test the assumption that the dispersion of the optic axial angle agrees with the equation:

$$\sin E = \sqrt{P + \frac{Q}{\lambda^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the values of P and Q for definite values of λ were obtained from the two sets of observation of E on the specimens Nos. 1 and 2, which are given in columns a and b in Table II. After reduction by the method of least squares, they are:

for a, $P_1 = 0{\cdot}078744$, $Q_1 = -18236{\cdot}4$
for b, $P_2 = 0{\cdot}059173$, $Q_2 = -17498{\cdot}2$

Table VII gives the measured and computed values for different values of λ, and the divergences between the observed and computed angles.

Table VII.

λ in μμ	E of No. 1 (curve a)			E of No. 2 (curve b)		
	Computed.	Observed.	Difference.	Computed.	Observed.	Difference.
478	—	0° 0′	—	—	—	—
481	0° 0′	—	—	—	—	—
486	2 14·7	2 55·0	− 40·3′	—	—	—
508·5	5 12·1	5 8·5	+ 3·6	—	—	—
535	7 2·5	6 53·5	+ 9·0	—	—	—
542	—	—	—	—	0° 0′	—
544	—	—	—	0° 0′	—	—
554	7 59·5	7 51·0	+ 8·5	2 40·3	2 49·7	− 9·4′
589	9 18·7	9 8·5	+ 10·2	5 21·7	5 16·2	+ 5·5
610	—	—	—	6 19·7	6 11·0	+ 8·7
644	10 44·8	10 41·5	+ 3·3	7 29·3	7 22·3	+ 7·0
671	11 16·6	11 9·0	+ 7·6	8 11·5	8 20·0	− 8·5

The observed and computed values, plotted on millimeter paper, are shown in Plate X, fig. 2, the abscissae measuring the wave-length in $\mu\mu$, and the ordinates E ($1° = 10$ mm.) being measured from a horizontal line supposed to pass through the acute bisectrix for all wave-lengths. It will be seen from the table and figure that the observed and computed values agree well together—the only serious divergence in E being that in the immediate neighbourhood of the point $E = 0°$. The zero-point of the computed ordinate is in each case displaced down the spectrum—to an extent of $3\,\mu\mu$ in curve a, and $2\,\mu\mu$ in curve b. With the exception of this portion of the curves, equation (1) may be accepted as correct.

Instead of equation (1) Pockels[1] uses:

$$\sin E = \frac{a}{\sqrt{\gamma^2 - a^2}} \sqrt{A_3 - A_2 + \frac{B_3 - B_2}{\lambda^2}}. \quad . \quad . \quad . \quad (1^*)$$

In obtaining this equation he assumes E to be always small, the fraction $\frac{a}{\sqrt{\gamma^2 - a^2}}$ to be constant, and that the dispersion of the refractive indices is given by

$$\mu_n^2 = A_n + \frac{B_n}{\lambda^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Equations (1) and (1*) are identical, if

$$\left.\begin{aligned} P &= \frac{a^2}{\gamma^2 - a^2}(A_3 - A_2) \\ Q &= \frac{a^2}{\gamma^2 - a^2}(B_3 - B_2) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To prove the truth of equations (3) in detail is not easy without very accurate data of the refractive indices. But the following results show that the variation of $\frac{a^2}{\gamma^2 - a^2}$ may be neglected in the determination of E.

Let $$\frac{a}{\sqrt{\gamma^2 - a^2}} = x \qquad \text{for } \lambda = 486\,\mu\mu,$$

and $$\frac{a_1}{\sqrt{\gamma_1^2 - a_1^2}} = x + \Delta x \text{ for } \lambda = 671\,\mu\mu;$$

these wave-lengths being the extremes taken in the series of the determinations of the refractive indices a and γ.

[1] F. Pockels, loc. cit., p. 71.

For $\lambda = 486\mu\mu$, $\quad \alpha = 1{\cdot}5265$, $\quad \gamma = 1{\cdot}5312$,

$$\therefore \quad x = 12{\cdot}78.$$

For $\lambda = 671\mu\mu$, $\quad \alpha_1 = 1{\cdot}5180$, $\quad \gamma_1 = 1{\cdot}5226$,

$$\therefore \quad x + \Delta x = 12{\cdot}91.$$

Hence $$\Delta x = 0{\cdot}13.$$

The change in x is therefore only 1/100 of its value.

Again, from the equation

$$\sin E = \alpha \sqrt{\frac{\gamma^2 - \beta^2}{\gamma^2 - \alpha^2}},$$

we obtain

$$\sin E' = x\sqrt{\gamma^2 - \beta^2} \qquad \text{for } \lambda = 486\mu\mu,$$

$$\sin E'' = (x + \Delta x)\sqrt{\gamma_1^{\,2} - \beta_1^{\,2}} \qquad \text{for } \lambda = 671\mu\mu.$$

Now put $$\sin E_0'' = x\sqrt{\gamma_1^{\,2} - \beta_1^{\,2}}.$$

$$\therefore \quad \sin E'' - \sin E_0'' = \Delta x \sqrt{\gamma_1^{\,2} - \beta_1^{\,2}}.$$

And writing ΔE for $E'' - E_0''$, and neglecting terms of the second order, we obtain

$$\Delta E = \frac{\Delta x \sqrt{\gamma_1^{\,2} - \beta_1^{\,2}}}{\cos E_0''}.$$

Introducing now the value of Δx, and that of $\sqrt{\gamma_1^{\,2} - \beta_1^{\,2}} = 0{\cdot}01483$, we get [1]

$$\Delta E = \frac{0{\cdot}13 \times 0{\cdot}01483}{\cos E_0''}.$$

When E_0'' is 10°, ΔE is less than 7′; and when $E_0'' = 15°$ it is less than 9′.

Hence the effect on E of the change in $\dfrac{\alpha}{\sqrt{\gamma^2 - \alpha^2}}$ is so small that it may be neglected, and it follows that P and Q of equation (1) may be replaced by their equivalents given in (3).

It is of interest to examine the change in the true optic axial angle, which is given by the equation:

$$\tan V = \frac{\alpha}{\gamma}\sqrt{\frac{\gamma^2 - \beta^2}{\beta^2 - \alpha^2}} \quad \left(\text{or, writing } z \text{ for } \frac{\alpha}{\gamma\sqrt{\beta^2 - \alpha^2}}\right)$$

$$= z\sqrt{\gamma^2 - \beta^2} \quad \ldots\ldots\ldots \quad (4)$$

[1] For $\lambda = 671\mu\mu$, $\gamma = 1{\cdot}52255$ and $\beta = 1{\cdot}52248$.

By computation, we find

for $\lambda = 486\mu\mu$, $z = 8{\cdot}349$,

and for $\lambda = 671\mu\mu$, $z + \Delta z = 8{\cdot}487$.

$$\therefore \quad \Delta z = 0{\cdot}138.$$

Now taking differences, we find from (4)

$$\Delta V = \Delta z \cos^2 V \sqrt{\gamma^2 - \beta^2}.$$

Now ΔV is greatest when $V = 0$, and $\lambda = 671\mu\mu$. Introducing the corresponding values, we find that

$$\Delta V < 0{\cdot}138 \times 0{\cdot}0148 < 8'.$$

Hence it follows that, within the stated range of the spectrum, the change in the factor $\frac{a}{\gamma\sqrt{\beta^2 - a^2}}$ produces no sensible effect in the angle V, and that the great dispersion of V results practically from the factor $\sqrt{\beta^2 - \gamma^2}$. The equation (4) may then be expressed by the dispersion formula:

$$\tan V = \sqrt{P_t + \frac{Q_t}{\lambda^2}}. \qquad \ldots \ldots \quad (5)$$

To test this transformation the values of V in specimen No. 1 were computed from β and the observed E, and are given in the second column of Table VIII. The coefficients P_t and Q_t were then computed in a manner similar to that by which P and Q of equation (1) were found. V' (given in column 3) was then determined by introducing into equation (5) their values, $P_t = 0{\cdot}0329$, and $Q_t = -7541{\cdot}2$.

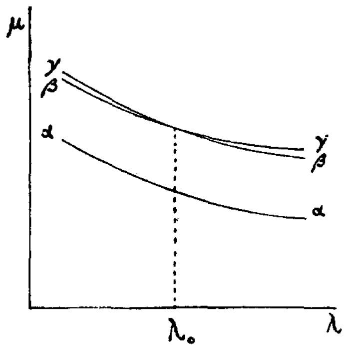
Table VIII.

λ in $\mu\mu$	V	V'	ΔV
478	0° 0'	—	—
478·9	—	0° 0'	—
486	1 54·3	1 47·0	+ 7·3'
508·5	3 22·2	3 29·7	– 7·5
535	4 30·25	4 37·6	– 7·35
589	5 58·7	6 1·8	– 3·1
644	6 59·7	6 55·0	+ 4·7
671	7 17·8	7 14·5	+ 3·3

These values are plotted in Plate X, fig. 2, on the same scale as the values of E. It is clear that the curves for both values agree very wel, and consequently equation (5) may also be employed to give the dispersion of the axial angle of the sanidine plate.

The Change in the Dispersion Formulae when E passes through Zero.

In the preceding discussion the plane of the optic axes was taken to be unchanged. When, however, E passes through zero, the plane changes through 90°, and β in the first axial plane becomes γ in s second position. In this new plane all that is needed to obtain the dispersion formulae for E and V is to change the signs of the coefficients P, Q and P_t, Q_t in equations (1) and (5).



Suppose the relative values of α, β, and γ for a crystal in which E passes through zero to be given by the curves in the accompanying text-figure, and λ_0 to be the wave-length for which $E = 0°$. At this point the plane of the optic axes turns through 90°, and the axis which was the mean principal axis becomes that of γ, whilst that corresponding to γ becomes the mean one. If now for values of $\lambda > \lambda_0$ the dispersion of the axial angle is represented by (1) and (5), then for values $\lambda < \lambda_0$ the dispersion is given by :—

$$\sin^2 E = -P - \frac{Q}{\lambda^2}, \quad \ldots \ldots \quad (1^*)$$

and

$$\tan^2 V = -P_t - \frac{Q_t}{\lambda^2}. \quad \ldots \ldots \quad (5^*)$$

To test this, the coefficients P and Q in equation (1*) were calculated from the observed values of E for different wave-lengths, the optic plane being parallel to that of symmetry. These observations were made in one and the same part of specimen No. 2, care being taken to avoid using a different portion in which the optic angle varied. The values obtained were :

$$P_3 = -0{\cdot}048170$$
$$Q_3 = 16410{\cdot}4,$$

where the suffix 3 is used to denote the coefficients of curve (*e*).

Hence for curve (*e*) lying in the symmetry-plane the signs of P_3 and Q_3 are the reverse of those of P_1, Q_1, and P_2, Q_2, the coefficients respectively of the curves (*a*) and (*b*) which lie in planes perpendicular to that of symmetry.

Table IX gives the observed E and its values computed from the above constants P_3 and Q_3. In Plate X, fig. 2, the observed values in curve (*e*) are shown by large circlets, those computed from the dispersion formula (1*) by smaller circlets.

Table IX.

E of No. 2. Axial Plane parallel to (010).

λ in μμ	Observed.	Calculated.	Difference.
486	8°28·3′	8°23·5′	+ 4·8′
508·5	7 4·0	7 6·3	− 2·3
535	5 26·3	5 29·5	− 3·2
554	4 4·8	4 10·5	− 5·7
575	—	2 11·7	—
584	—	0 0	—
585	0 0	—	—

It will be seen that the computed and observed values agree together very well, and so far justify the statement that in the second axial plane the signs of the coefficients are the reverse of those in the first.

There is some interest in seeing how the curves (*b*) and (*e*) would be continued in the plane at right angles to that in which the observations were actually made. The continuations obtained by changing the signs of P_2, Q_2 and P_3, Q_3 are shown in curves (*f*) and (*g*) of Plate X, fig. 2, and the computed values at the points indicated by circlets are given in Table X. There is a want of congruence at the point $E = 0°$ due to the divergence of the actual and computed points of uniaxiality already mentioned on p. 247.

Table X.

E in the Rotated Planes computed by changing the signs of the Coefficients of Curves (b) and (c).

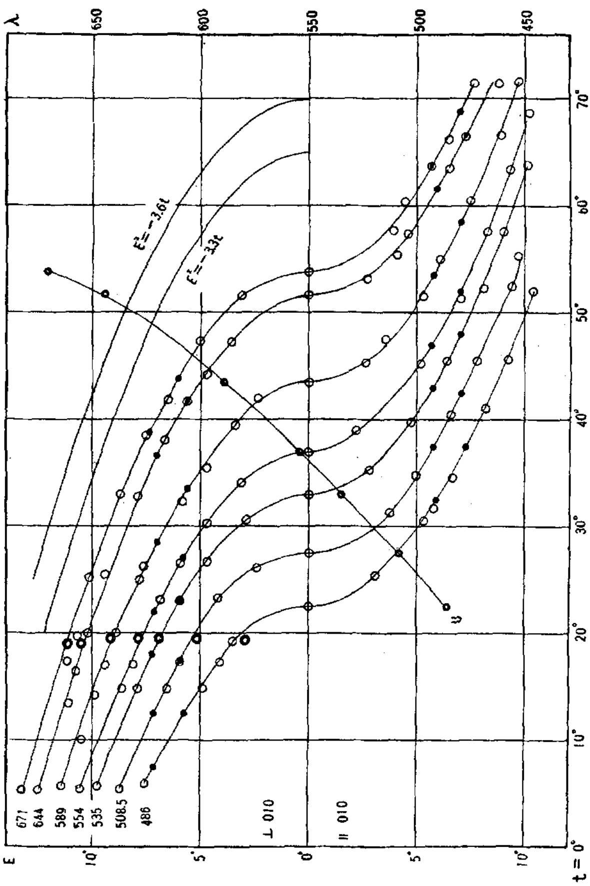
λ in μμ	Curve *f*	Curve *g*
	A. P. ∥ (010)	A. P. ⊥ (010)
486	7° 0·8′	—
508·5	5 17·3	—
535	2 32·2	—
544	0 0	—
583·6	—	0° 0′
589	—	1 41·2
610	—	3 39·3
644	—	5 19·2
671	—	6 12·8

Explanation of Plates.

Plate IX.—The abscissae give temperature, and ordinates on the left measured up and down from the zero-ordinate give the angle E. The simple circlets mark direct observations for the wave-lengths placed at one end of the curve on which they lie; the double circlets give direct observations without a heating cell. The black dots give the positions on each curve for temperatures differing from that of the zero-point by 10° and 15°. The parabolic curves above the zero-line give angles when the axial plane is perpendicular to that of symmetry, the lower parabolic portions give the angles when the axial and symmetry planes coincide. The parabolas without dots or circlets are the limiting ones described in the text (p. 242). The curve U gives the relation between temperature and wave-length at which the crystal becomes uniaxial—the wave-lengths in μμ on the ordinates shown by numbers on the right.

Plate X, fig. 1, gives the curves for equal temperatures, the abscissae measuring wave-lengths in μμ, and the ordinates the angle in air measuring from the zero-line, the curves above and below it being in the planes at right angles as indicated. The crosses give the points on the curves differing by equal differences of abscissae—25, 50, and 75 in μμ—from the zero-point.

Plate X, fig. 2, are $E\lambda$ curves similar to those of fig. 1, except that the black dots and double circlets give the ordinates of V. The meaning of the circlets is given in the diagram.



S. KÔZU: OPTIC AXIAL ANGLE OF SANIDINE.

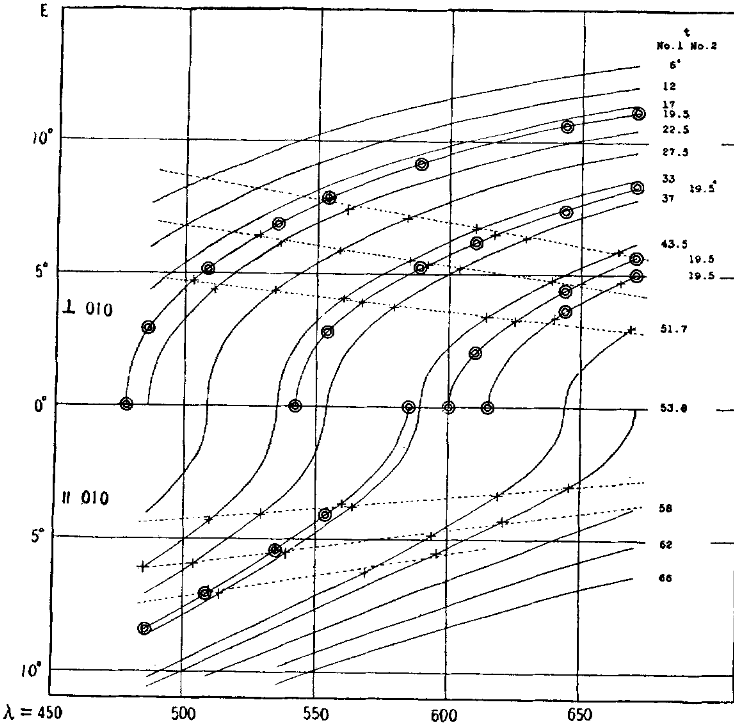


Fig. 1.

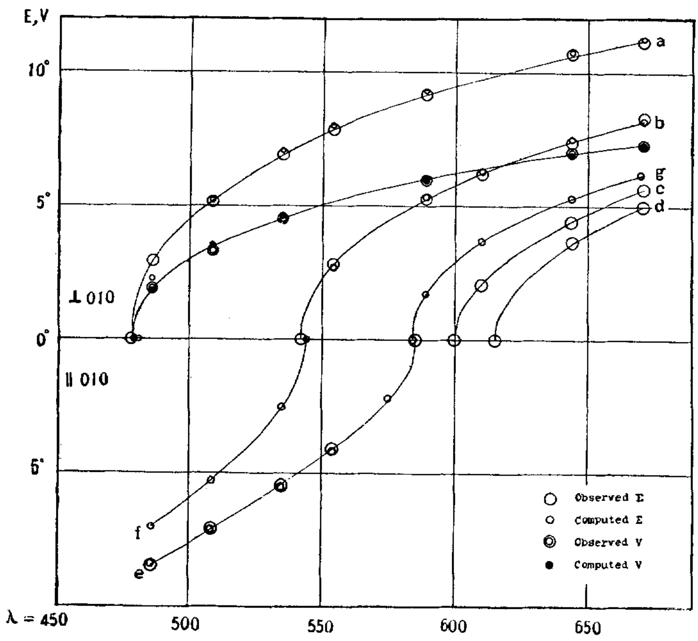


Fig. 2.

S. KÔZU: OPTIC AXIAL ANGLE OF SANIDINE