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The frequency-distribution of igneous rocks.

I. Frequency-distribution of the major oxides in analyses of igneous rocks.

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1. *Introduction.*

THE quantitative distribution of igneous rocks has recently assumed a place of importance in petrogenic discussions. This is due largely to the influence of Daly,[1] who made a survey of the areal distribution of rock types as shown by their outcrops on published maps, and reached the well-known conclusion that intrusions are overwhelmingly granitic in composition, and that extrusions are largely basaltic. If further research should firmly establish this result, it will unquestionably be a fact of the first order of importance of which any general theory of petrogenesis must take cognizance.

[1] R. A. Daly, Igneous Rocks and their Origin, New York and London, 1914, chap. iii.

Collected analyses of the world's rocks should, of course, reflect their distribution, but Daly and others have pointed out that, owing chiefly to the tendency of petrologists in the past to neglect the commoner and pay great attention to the rarer types, the frequency shown by analyses departs from that shown by the rocks themselves, especially in respect to alkaline rock types which are more abundantly represented in analyses than in nature. It has, therefore, been urged that, in taking out averages, analyses of rocks should be weighted according to areas occupied. This suggestion, however, Washington[1] refused to carry out, partly on the ground that as material accumulates the distribution must necessarily more and more represent the facts of natural distribution; and partly because any system of weights introduces a subjective element, very difficult to evaluate, into the calculations. Moreover, since analyses are now chiefly made as part of the routine of official surveys, the personal effect of selection is much reduced.

The first object of this paper is, accordingly, to determine the frequency-distribution of all the major oxides in Washington's latest (1917) collection of superior analyses of igneous rocks.[2] This determination will afford an immediate test of the relative abundance of chemical types. At the same time the data will give a means of calculating approximately the probable composition of the most frequent rock types. Further, Dr. Harker fortunately investigated the frequency-distribution of silica in the earlier (1903) collection by Washington,[3] and a comparison of this with the new distribution of silica will enable the general trend of the changes to be gathered, and form a basis upon which judgement as to the stability of the distribution can be made.

In Part II of the present paper the actual frequency-curves will in the first place be analysed with a view to discovering the laws of distribution. Secondly, an inquiry will be made into the types of frequency-distribution to be expected of the various processes that have been appealed to as causes of rock variation. And, finally, general theories of petrogenesis will be critically examined from the standpoint of competence to produce the actual distribution of igneous rocks.

[1] H. S. Washington, The chemistry of the earth's crust, Journ. Franklin Inst., 1920, vol. 190, p. 770. [Min. Abstr., vol. 1, p. 160.]

[2] H. S. Washington, Chemical analyses of igneous rocks, published from 1884 to 1913. U.S. Geol. Survey, 1917, Prof. paper 99.

[3] H. S. Washington, ibid., published from 1884 to 1900. U.S. Geol. Survey, 1903, Prof. paper 14.

2. *Averages.*

Before stating the results of the investigations, it is desirable, owing to a certain confusion in nomenclature, to draw attention to the names and properties of the various averages employed by statisticians as standards of comparison. The chief are the *arithmetic mean*, the *median*, and the *mode*. Besides these the geometric and harmonic means have a limited usefulness for certain special purposes.

The *arithmetic mean* is readily understood, easily calculated, uses all the data, and, moreover, lends itself far better to algebraic treatment than the other kinds of averages. However, in the case of a discontinuous series, it does not necessarily correspond to an actual case. Thus, if the values averaged all lie either between 0 and 5, or between 10 and 15, the arithmetic mean may easily fall within the unoccupied region from 5 to 10. Nevertheless, the arithmetic mean is the best average for general purposes.

If all values are arranged in order of magnitude, then the *median* is the central or middlemost value. It is easily calculated, but may be totally removed from the type. It is, however, applicable to quantities which are not capable of measurement, and is chiefly useful in such cases.

The *mode* may be defined as the most frequently occurring value, or as the centre of density of the values. It may obviously furnish exceedingly useful information—in petrology, for example, the igneous rock most frequently met with is for many purposes more important than the mean rock. The mode is also the most stable of the averages, and is least affected by extremes. If, for example, the values of an oxide in analyses lie mainly between 0 and 7 per cent., the occurrence of a single value at 50 per cent. will displace the mean without affecting the mode at all. It is, however, in many cases less easy to calculate than the mean, and its position may often be very indefinite.

The term 'mode' was introduced into statistics by Karl Pearson in 1895.[1] It is unfortunate, therefore, that American petrologists in 1902[2] reintroduced the term with an entirely new meaning, defining it as the actual quantitative mineral composition of an igneous rock. The word 'mode' might, therefore, appear in petrology meaning either the most frequent rock, or some particular rock. Doubtless a convenient

[1] Karl Pearson, Phil. Trans. Roy. Soc., Ser. A, 1895, vol. 186, p. 343.

[2] Cross, Iddings, Pirsson, and Washington, Journ. Geol. Chicago, 1902, vol. 10, p. 555.

term to express actual mineral composition as opposed to calculated mineral composition is highly desirable; but, since statistical methods become more and more useful with the accumulation of numerical data, the term 'mode' must be retained with its prior and statistical significance. And, naturally, it is used in that sense throughout this paper.

3. *The Frequency-Distribution of Silica.*

In order to construct frequency curves and determine the mode, all values falling within the limits of a chosen interval are counted and

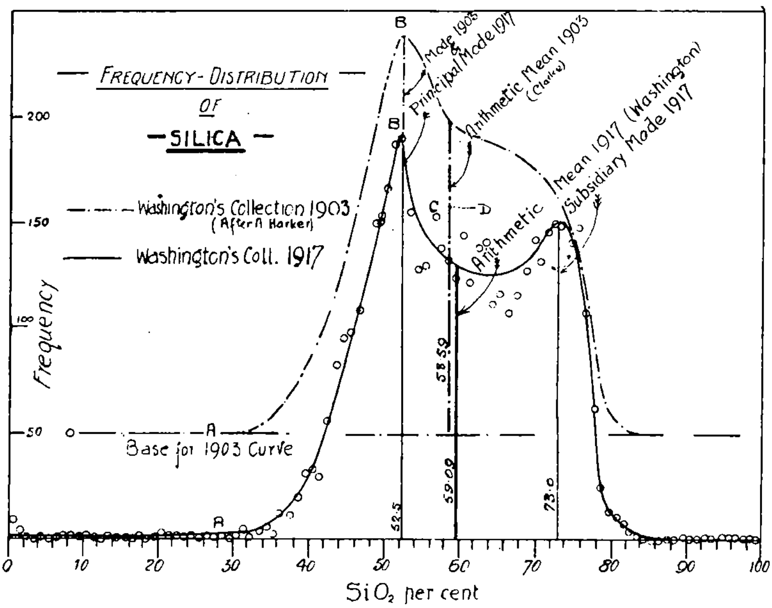

FIG. 1.—Comparison of the frequency-distribution of igneous rocks as shown by silica percentage in H. S. Washington's collections of analyses of 1903 and 1917.

tabulated. The interval employed in this investigation was one per cent. So small an interval is justified by the extensive data available, and, moreover, somewhat simplified the counting. All values of an oxide, then, occurring from 0 but below 1·0 per cent. are tabulated and plotted as 0·5 per cent.; those falling in the interval from 1·0 but below 2·0 are tabulated and plotted as 1·5; and so on.

The results obtained by counting the silica percentages in this way are plotted on fig. 1. The actual figures for each interval are shown by small circles, and the full line is the link polygon 'smoothed' by eye.

A study of this curve reveals several rather striking characters in the frequency-distribution. In the first place the curve rises steeply at the basic and acid ends—the latter being the steeper. The curve flattens out rapidly at both ends, and the points in these regions lie closely on a smooth curve. There are, secondly, two peaks indicating a principal basic mode at 52·5, and a subordinate acid mode at 73. The basic mode is, moreover, more sharply defined than the acid. Thirdly, there is a depression between the two peaks. This is still distinct even if the curve were drawn through the maximum points instead of averaging the observations; and, moreover, it deepens as it approaches the acid mode.

It should also be noted that it is over the range between the two modes that marked oscillations in frequency occur. However, if the high values from *C–D* are neglected, the points are fairly consistent and give a very deep valley, approaching a minimum at 68 per cent. Now the range *C–D* (from 56 to 66 per cent.) is just the region into which most of the extreme intermediate rocks fall. There is, also, a sharp rise near zero owing to the inclusion of a number of magmatic iron-ores in the analyses.

It is therefore seen that the frequency-distribution of silica, although not exactly irregular, does not conform to any simple type.[1]

This frequency-distribution may now be compared with that of the earlier (1903) collection of Washington as obtained by Dr. Harker,[2] whose curve is re-drawn in fig. 1 for this purpose to the same scale as the 1917 curve, but the x-axis is lifted slightly to avoid confusion. Some features remain quite unchanged, but in other respects there is noteworthy divergence. The basic slope and the lower part of the acid slope in the two distributions are practically the same, or in other words the extreme portions of both curves are nearly coincident. Moreover, the 1903 curve shows only one peak, which is remarkable, for it falls *exactly in the same position as the principal* (*basic*) *mode of the new distribution* (52·5 per cent.). Further, there has only been a slight change in the arithmetic means—that for 1903 is shown by the chain-dotted ordinate, and that for 1917[3] by the thick full ordinate

[1] For types of distribution the reader may be referred to U. Yule, Introduction to the theory of statistics, London, 1919, chap. vi.

[2] A. Harker, Natural History of Igneous Rocks, London, 1909, p. 148.

[3] H. S. Washington, Journ. Franklin Inst., 1920, vol. 190, p. 778.

—the shift being about 1 per cent. to the right in the direction predicted by Dr. Harker.

The changes then that have taken place all lie in the central portion of the curve. There has been a steepening of the upper part of the acid end; the development of a new peak at 73; and the appearance of the deep depression in the intermediate zone. Indeed there is some stability in the main features and the changes seem to be taking place in definite directions.

4. *The Frequency-Distribution of other Major Oxides.*

The frequencies of the other major oxides are plotted in fig. 2. Only the curves for alumina and soda are smoothed, the remainder being drawn as link polygons. In the latter the intersections of straight-line portions mark the locations of the actual observations. In all cases, however, smooth curves can be drawn fitting the points as closely as in the case of soda and alumina. Whilst these curves are on the whole less interesting than that for silica, nevertheless, they have distinctive features and yield useful information, and may now be briefly considered.

Alumina shows a good horizontal range, i. e. has a wide 'dispersion'. Only one peak occurs, and there is not the slightest indication of the existence of a second. This peak shows that the mode is 16, and the distribution is nearly symmetrical about it.

The curves for *ferrous and ferric oxides* are very similar. They are both asymmetrical, and both show one mode at 1·5 per cent. The curve for ferrous oxide is perhaps rather more asymmetrical, but the main difference lies in the greater dispersion shown by this curve.

Magnesia possesses the most extreme type of asymmetrical distribution, known as the J-type. The curve appears to be asymptotic to both axes, and the mode is near the zero. Thus the great majority of rocks have a low magnesia content, and only a very few have much.

Lime also has the J-type of distribution, which is, however, modified by the appearance of a second peak. Accordingly lime has two modes —one near zero and the other, due to the influence of the basic end, at 7·5.

Potash gives another curve exhibiting two peaks. Here, however, the asymmetry is less extreme, and the potash curve resembles that of silica more closely, perhaps, than that of any other oxide. The principal mode is at 1·5; and the subordinate one at 4·5 is chiefly due to the influence of intermediate and acid rocks.

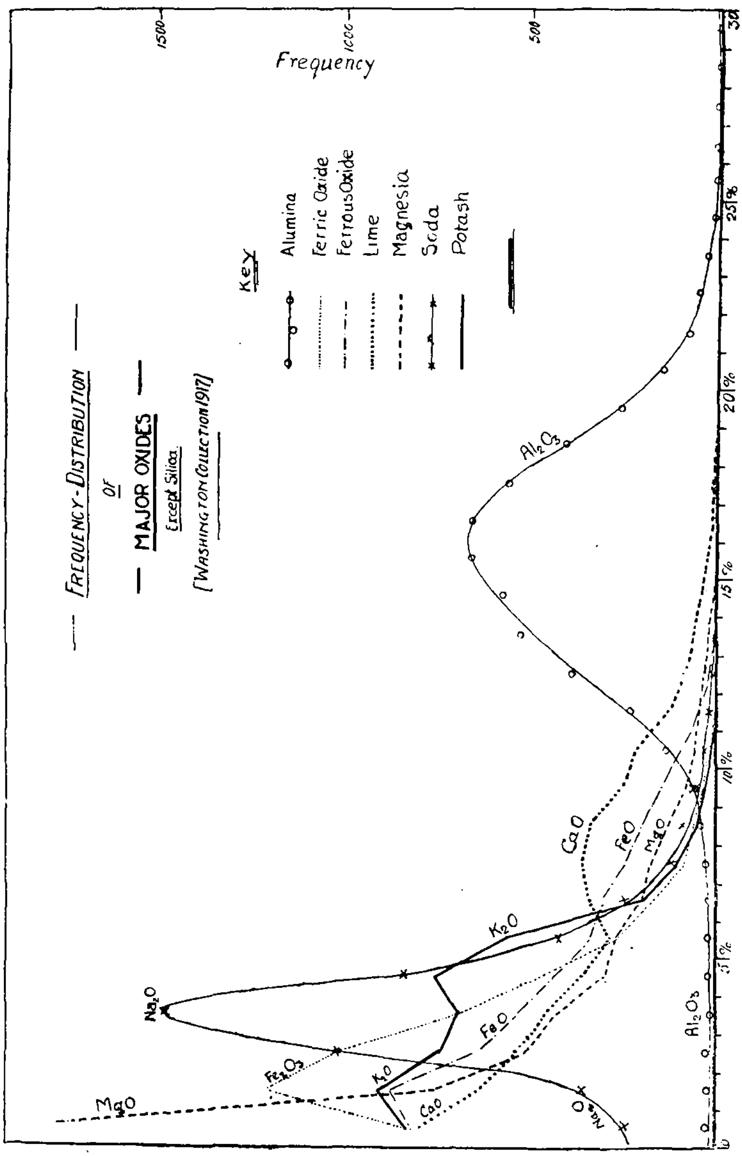

Fig. 2.—Frequency-distribution of oxides other than silica.

The *Soda* curve shows one mode only, and is rather more symmetrical than potash.

It is, therefore, apparent that all these oxides tend to have asymmetrical distributions, most extreme in the case of lime and magnesia and least so in alumina. Two of the oxides—lime and potash—resemble silica in the possession of two modes, but on the whole the modes occupy low positions because the amount of any base may drop to zero at all parts of the silica scale. Moreover, silica has by far the greatest dispersion, and with the single exception of alumina the other oxides are crowded towards the zero end.

That there is some abundance of alkaline rocks in the collection of analyses is not only indicated on the silica diagram, but is also confirmed by the alkali curves. The mode for soda indicates an abundance of moderately sodic rocks, and if the alkalis had been added together the combined mode would be found considerably to the right, located perhaps near 6 per cent. or even higher. It may, therefore, be admitted that Daly's contention that collected analyses embrace a considerable number of alkaline rocks is still justified. On the other hand, the subordinate mode for lime indicates that calcic rocks are also well represented.

Finally, it may be pointed out that these diagrams show in a graphic way the advantage possessed by silica as a basis for the classification of igneous rocks, partly on account of the striking characters of the distribution, but chiefly because of its wide dispersion as compared with that of other oxides.

5. *The Most Frequent Igneous Rock Types.*

As a first step towards finding from the analytical data what are the most frequent types of igneous rocks, the modes of the oxides are collected in Table I under two headings corresponding to the two silica modes. A third column, containing Washington's recently calculated mean, is added.[1] Now it is not possible to allot the modes appropriately to the acid and basic ends by mere inspection of the diagrams, since they are not constructed on a common basis, but the general position was easily noted during counting. Where an oxide exhibits but one mode it means that both the acid and basic regions contributed, and the value is accordingly entered in each column. Where there are two modes they are assigned as determined during counting.

[1] H. S. Washington, Journ. Franklin Inst., 1920, vol. 190, p. 773.

Table I. The Modes and the Mean of Igneous Rock Analyses.

Constituent.	Basic Mode.	Acid Mode.	Arithmetic Mean (H. S. Washington, 1920).
Silica	52·5	73·0	59·09
Alumina	16·0	16·0	15·35
Ferric Oxide	1·5	1·5	3·08
Ferrous Oxide	1·5	1·5	3·80
Magnesia	0·5	0·5	3·49
Lime	7·5	0·5	5·08
Soda	3·5	3·5	3·84
Potash	1·5	4·5	3·13
Totals	84·5	101·0	96·86

It is not of course to be expected that the modal totals will add up to 100, for the following reasons:

1. All the constituents are not taken into account, and the remainder judging by the arithmetic mean amount to about 3 per cent.

2. Since the distributions do not conform to any simple type, the different modes are largely independent, their position having been determined by complex causes. Hence the mode of igneous rocks as a unit cannot be directly determined from the modes of its constituents.

3. This failure of summation may be connected indirectly with the bipolarity of the silica, which forms the bulk of most rocks. The high value of silica at the acid pole imposes restrictions on the amount of variation possible to other oxides; whilst the lower values at the basic pole permit of much greater fluctuation in other oxides.

Thus both the basic and acid suites contribute many low values to all oxides, and, therefore, as already mentioned the modes are segregated towards the origin. Alumina seems to be equally influenced by both poles, but lime and magnesia at the basic end fluctuate inversely. A better approach, therefore, to their most frequent values at the basic end would have been attained by investigating the distribution of their sum. Again the higher mode of potash is due to contributions from all parts of the silica scale. It has most right perhaps to be placed in the acid column, although it is not the most frequent value of potash for rocks with a silica content of 73 per cent.

Owing to these conditions the basic summation is very low and the acid a little high, as might be expected from the above discussion. Now the nature of a mode does not permit of adjustment to a theoretical summation. However, since silica is the best determinant of the position of a rock in the classification, the requirements of practice will be satisfied if we can find what is the most common composition of rocks possessing 52·5 and 73 per cent. of silica respectively. This result could be most readily obtained if variation diagrams of the oxides plotted against silica were available, for the centres of densities of the points at the ordinates 52·5 and 73 would give the required values. Curves of this sort were prepared[1] from the analyses collected by Iddings—a collection that gave a frequency-distribution for silica of the same type as the 1917 collection of Washington. The values obtained from these curves are presented in Table II, and here again the greater vertical dispersion renders the basic values a little less determinate than those at the acid end.

Table II. Composition of the Most Frequent Rock at the Silica Poles.

Constituent.	Basic.	Acid.
Silica	52.5 ...	73·0
Alumina	16·0 ...	14·0
Ferric Oxide / Ferrous Oxide }	10·0 ...	1·5
Magnesia	6·0 ...	0·2
Lime	9·0 ...	0·5
Soda	3·0 ...	4·0
Potash...	1·0 ...	3·5
Totals	97·5 ...	96·7

Now since it is undesirable to attempt to adjust these figures in any way, the rock type represented was determined, not by calculating the 'norm', but by comparison with the averages of Daly. The basic member is a somewhat acid olivine-dolerite, and the acid member is a rather typical alkali-granite. Washington, it may be mentioned, determined that the arithmetic mean of the analyses represented a granodiorite or an augite-andesite.

Now it will be evident that we have arrived at the same result as Daly did by measurement of the surface areas of igneous rock outcrops. His own words are 'quantitatively considered, the igneous rocks of the globe chiefly belong to two types, granite and basalt'.[2] Further, summing his areas for North America, we find that rocks from granite to diorite (including rhyolites, &c.) occupy an area of 8,300 square miles; and those from augite-andesite to basalt (including gabbros, &c.) an area of 8,400 square miles. These figures give a ratio of acid to basic of

[1] The alumina curve was published in Min. Mag., 1921, vol. 19, p. 131, fig. 1.
[2] R. A. Daly, loc. cit., p. 52.

1 : 1·1. Now the rocks totalled above are those occurring close to the maximum ordinates of the silica curve, and the ratio of these ordinates, acid to basic, is 1 : 1·3.

Agreement, therefore, as to the prevalent types of igneous rocks obtained by two very different methods is surprisingly close. On the one hand Daly's views as to distribution of types are supported; and, on the other, the trend of the changes in the new collection of analyses justifies Washington's refusal to weight the averages according to area. It should also be noted that the position of the mode is not affected by the inclusion at any par of the range of a moderate number of types in excess of their natural frequency.