The graphical determination of the constants of a shear.

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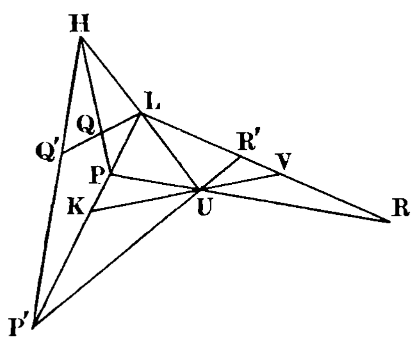
RECENTLY A. Johnsen[1] has given a graphical solution by means of the stereographic projection of the problem of determining the constants of a homogeneous shear of a crystal, knowing the initial and final indices of two faces or two edges.

The same problem may be solved more easily by means of the gnomonic projection.

In a shear there are two fixed lines l, m intersecting at right angles at O, such that each particle of the crystal moves parallel to l through



a length proportional to its distance from the plane lm. The shear is considered given when we know l, m, and the line through O perpendicular to m making supplementary angles with l before and after the shear.

Suppose we form the gnomonic projection of the face-poles and zones of the crystal from a sphere with centre O. Let V be the centre of

[1] A. Johnsen, Neues Jahrb. Min., 1921, vol. ii, p. 1 [Min. Abstr., vol. 1, p. 220].

the gnomonic projection, i. e. the point where the plane of the projection Π touches the sphere. Suppose also we know the intersections (P, Q before shearing and P', Q' after) with Π of the lines through O parallel to two edges of the crystal.

Let PP' and QQ' meet in L; PQ and $P'Q'$ meet in H.

Let the line joining H to the intersection of PQ' and $P'Q$ meet PP' in K. Let VK meet LH in U. Let PU, $P'U$ meet LV in R, R'. Then L is the intersection of l with Π, LH is the intersection of the plane lm with Π, and OR is zone-axis brought to OR' by the shear, such that OR and OR' are perpendicular to m and make supplementary angles with l.

For the proof of the construction we notice that (1) it is a construction which is not altered by ordinary conical (plane-to-plane) projection; (2) a change of the plane of gnomonic projection is equivalent to a conical projection; (3) the construction is evidently correct when Π is parallel to the plane lm, for then PP', QQ', RR' are equal and parallel.

If the initial and final projections A and A', B and B' of two face-poles are given, instead of the intersections with Π of lines through O parallel to two edges, then AA', BB' meet in the intersection with Π of the line through O perpendicular to the plane lm.

Knowing V, we get thus the line LH. Moreover, the initial and final positions AB and $A'B'$ of a zone are known, so that we can get L and proceed as before.