Optical dispersion of three intermediate plagioclases.

By SEITARÔ TSUBOI,

Of the Geological Institute, Imperial University of Tokyo, Japan.

[Read June 19, 1923.]

INTRODUCTION.

THE optical dispersion phenomena of felspars at the alkalic and calcic ends of the plagioclase series have been investigated by C. Viola [1] and S. Kôzu,[2] but for those of intermediate composition the data have been lacking. In view of the usefulness of the dispersion data of felspars for their determination in cleavage-flakes by the method to be described in the next paper, the present writer has made a study of the dispersion of certain intermediate plagioclases. The materials examined are:

(1) Oligoclase from Hawke mine, Bakersville, North Carolina.
(2) Andesine from Maeyama, Shinano, Japan.
(3) Labradorite from St. John's Point, County Down, Ireland.

These were put at the disposal of the writer by Professor W. J. Lewis and Dr. A. Hutchinson, of the University of Cambridge, England. My hearty thanks are due to them for this as well as for their great kindness in granting me the use of the Mineralogical Laboratory and also for constant interest and help throughout the work.

I. OLIGOCLASE.

The material used was a large cleavage mass of oligoclase, about 5 cm. across, from Hawke mine, Bakersville, North Carolina. It is highly fresh and homogeneous, and is very suitable for optical and chemical investigations. The oligoclase here treated of belongs no doubt to a species distinct from the felspars 'from Bakersville, North Carolina'

[1] C. Viola, Zeits. Kryst. Min., 1899, vol. 30, p. 437; 1900, vol. 32, p. 305.

[2] S. Kôzu, Sci. Rep. Tôhoku Imp. Univ. Sendai, Japan, 1914, ser. 2, vol. 2, p. 7; Min. Mag., 1915, vol. 17, p. 189.

investigated by A. Offret,[1] C. Viola,[2] and H. Tertsch,[3] the present one being higher in refractive indices.

Chemical Composition.

A chemical analysis made by the writer gave the result as shown in column I of Table I.

TABLE I.—Chemical Composition of Oligoclase.

	I.	II.
SiO_2	61·70	62·34 %
Al_2O_3	23·99	23·74
$Fe_2O_3 + FeO$ (as Fe_2O_3) ...	0.43	—
MgO	0·40	—
CaO	5·09	5·04
Na_2O	8·81	8.72
K_2O	0·17	0·16
Total	100·59	100·00

From the analysis in column I, assuming all of the CaO, Na_2O, and K_2O to enter into felspar molecules and neglecting Fe_2O_3 and MgO, the composition of the mineral is represented as $Or_{0\cdot9}\,Ab_{75\cdot1}\,An_{24\cdot0}$, the theoretical percentages for which are given in column II of the above table.

Refractive Indices and Birefringencies.

The principal refractive indices of the oligoclase for light of different wave-lengths were determined on a carefully polished plane surface by means of the Abbé-Pulfrich refractometer, the constants of which were determined by S. Kôzu.[4] A Hilger's wave-length spectrometer was used to get light of various colours from a Nernst lamp. The readings of wave-lengths on the spectrometer were checked by employing the bright lines of lithium (671 $\mu\mu$), sodium (589·3 $\mu\mu$), and thallium (535 $\mu\mu$).

The results of the measurements are given in Table II. Each of the values in the table was calculated from the mean of four readings of the critical angle corresponding to a principal refractive index, taken at two

[1] A. Offret, Bull. Soc. Franç. Min., 1890, vol. 13, p. 648.

[2] C. Viola, Zeits. Kryst. Min., 1900, vol. 32, p. 333.

[3] H. Tertsch, Tscherm. Min. Petr. Mitt., 1903, vol. 22, p. 159.

	α_D.	β_D.	γ_D.
Offret... ...	1.5389	1.5431	1·5469
Viola... ...	1·52898	1·53290	1·53915
Tertsch ...	1.5388	1·5428	1.5463

[4] S. Kôzu, Min. Mag., 1916, vol. 17, p. 255.

azimuths opposite to each other, with the telescope first on one side and then on the other. Each reading of the critical angle was taken to the tenth of a minute.

TABLE II.—Refractive Indices of Oligoclase. (Temperature 20·5° – 21° C.)

Wave-lengths (in $\mu\mu$).	α.	β.	γ.
700	1·5366	1·5408	1·5442
671	1·5373	1·5416	1·5450
644	1·5383	1·5424	1·5460
610	1·5395	1·5438	1·5472
589·3	1·5403	1·5447	1·5481
554	1·5421	1·5464	1·5497
535	1·5431	1·5475	1·5508
527	1·5437	1·5480	1·5513
508·5	1·5450	1·5493	1·5525

TABLE III.—Birefringencies of Oligoclase. (Temperature 20·5° – 21° C.)

Wave-lengths (in $\mu\mu$).	$\gamma - \alpha$.	$\gamma - \beta$.	$\beta - \alpha$.
700	0·0076	0·0034	0·0042
671	0·0077	0·0034	0·0043
644	0·0077	0·0036	0·0041
610	0·0077	0·0034	0·0043
589·3	0·0078	0·0034	0·0044
554	0·0076	0·0033	0·0043
535	0·0077	0·0033	0·0044
527	0·0076	0·0033	0·0043
508·5	0·0075	0·0032	0·0043

Positions of the Optic Axes.

To find the positions of the optic axes, A and B, of the oligoclase, two sections, s_1 and s_2, were prepared, one of which (s_1) showed the emergence of A, and the other (s_2) that of B, in the field of the conoscope. The conoscopic figures of these sections were so distinct that the points of emergence of the optic axes could be very accurately determined by means of them.

Position of the Optic Axis A.—The orientation of the section s_1 was determined with a reflecting goniometer to be as shown in figs. 1 and 2,[1] where

$b'\,(0\bar{1}0) : c\,(001)$	...	93° 45·5′
$b'\,(0\bar{1}0) : s_1$	...	39 14
$c\,(100) : s_1$	...	97 45

[1] In some of the following figures certain points have been displaced a little from their true positions in order to make the figures clearer.

Observing this section under a conoscope, the apparent central distance of the point of emergence of A was measured with a micrometer ocular. The readings for it were taken at various positions of zero-isogyre, so as to eliminate the possible error due to the shape of the interference-figure. From the result of this measurement the central angular distance of A within the crystal (ρ or $s_1 A$ in fig. 1) was calculated by means of Mallard's formula, using the values of β already given. Then the position of the point of emergence of A was determined with a co-ordinate micrometer

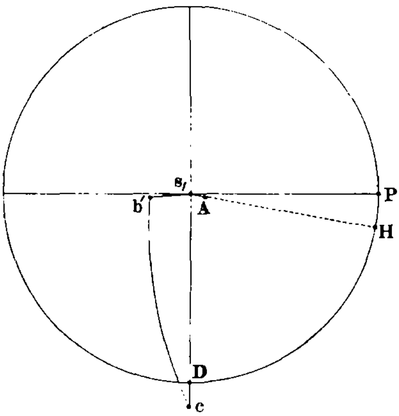

FIG. 1.

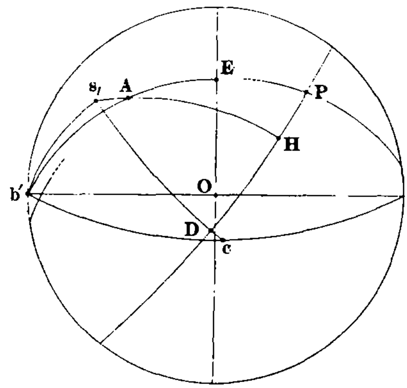

FIG. 2.

FIG. 1. Stereogram showing the positions of s_1 and A of oligoclase; projected on the plane of s_1. $s_1 A = \rho$, $PH = \theta$.

FIG. 2. Stereogram showing the positions of s_1 and A of oligoclase and the relations between ρ, θ, ϕ, and λ; projected on the plane perpendicular to the c-axis. $s_1 A = \rho$, $PH = \theta$, $EA = \phi$ (−), $OE = \lambda$.

ocular, taking the trace of $c(001)$ as abscissa, and from this the azimuth of A (θ or PH) with the trace of $c(001)$ was calculated. Table IV shows ρ and θ thus obtained for light of five different wave-lengths.

TABLE IV.

Wave-lengths (in $\mu\mu$).	ρ.	θ.
700	10° 19′	9° 0′
644	10 34	9 13
589·3	10 51	9 40
535	11 8	10 8
508·5	11 15	10 31

The values[1] of ϕ and λ for A can be found from those of ρ and θ in the following way:

In the spherical triangle $b's_1A$ (fig. 2) we have

$$\cos b'A = \cos b's_1 \cos s_1A + \sin b's_1 \sin s_1A \cos b's_1A \quad \text{..............(1)}$$

and $$\sin s_1b'A = \frac{\sin s_1A \sin b's_1A}{\sin b'A} \quad \text{..............(2)}$$

By solving the spherical triangle $b'cs_1$ whose three sides are known we obtain

$$b's_1c = 86° 26{\cdot}4' \quad \text{and} \quad cb's_1 = 97° 39{\cdot}2';$$

and we have also $$cs_1A = 90° - \theta,$$

$$b's_1A = b's_1c + cs_1A = 86° 26{\cdot}4' + 90° - \theta,$$

$$b'A = 90° + \phi, \qquad s_1A = \rho, \qquad b's_1 = 39° 14',$$

and[2]

$$s_1b'A = cb's_1 - (cb'O + Ob'A) = 97° 39{\cdot}2' - (\angle\beta - 90° + \lambda)$$
$$= 71° 15{\cdot}6' - \lambda.$$

Hence

$$-\sin\phi = \cos 39° 14' \cos\rho - \sin 39° 14' \sin\rho \sin(86° 26{\cdot}4' - \theta) \quad \text{......(3)}$$

and $$\sin(71° 15{\cdot}6' - \lambda) = \frac{\sin\rho \cos(86° 26{\cdot}4' - \theta)}{\cos\phi} \quad \text{..............(4)}$$

ϕ and λ for the optic axis A calculated by (3) and (4) are tabulated below (Table V).

TABLE V.—Position of the Optic Axis A of Oligoclase. (Temperature 21° C.)

Wave-lengths (in $\mu\mu$).	ϕ.	λ.
700	− 40° 39′	+ 68° 19′
644	− 40 25	+ 68 12
589·3	− 40 9	+ 68 2
535	− 39 54	+ 67 51
508·5	− 39 47	+ 67 43

Position of the Optic Axis B.—The position of the optic axis B of the oligoclase was determined with the section s_2, in the same way as above. The section s_2 was orientated as shown in figs. 3 and 4, where

b (010) : c (001)	...	86° 14·5′
b (010) : s_2	...	40 41
c (001) : s_2	...	86 39·5

[1] ϕ and λ are respectively the latitude and the longitude of the optic axis in the spherical projection, taking the great circle parallel to b (010) as the equator, and the meridian of the c-axis as the initial meridian (cf. fig. 2). In fig. 2, ϕ is positive when measured to the right (i.e. in the first and fourth quadrants); and λ is positive when measured upward (i.e. in the first and second quadrants).

[2] $\angle\beta$, the angle between the crystallographic axes $c : a$, for $Ab_{76}\,An_{24}$ is 116° 23·6′ according to E. Schmidt, Die Winkel der kristallographischen Achsen

The values of ρ' and θ' for B are given in Table VI.[1]

TABLE VI.

Wave-lengths (in $\mu\mu$).	ρ'.	θ'.
700	6° 53′	9° 28′
644	6 56	8 24
589·3	6 59	7 8
535	7 4	5 29
508·5	7 8	4 46

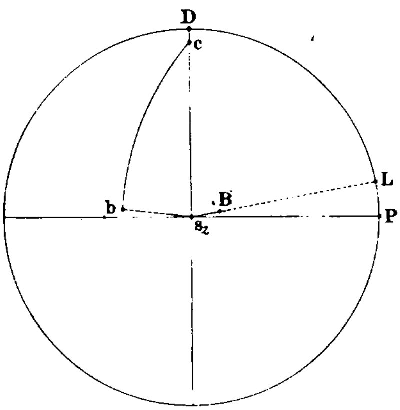

FIG. 3.

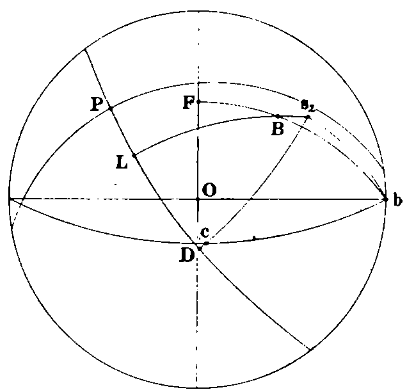

FIG. 4.

FIG. 3. Stereogram showing the positions of s_2 and B of oligoclase; projected on the plane of s_2. $s_2B = \rho'$, $PL = \theta'$.

FIG. 4. Stereogram showing the positions of s_2 and B of oligoclase and the relations between ρ', θ', ϕ', and λ'; projected on the plane perpendicular to the c-axis. $s_2B = \rho'$, $PL = \theta'$, $FB = \phi'$, $OF = \lambda'$.

From the spherical triangle bcs_2 (fig. 4) we have

$$bs_2c = 88°\ 7{\cdot}3' \quad \text{and} \quad cbs_2 = 89°\ 14{\cdot}6'.$$

We have also $\quad cs_2B = 90° - \theta'$,

$$bs_2B = 88°\ 7{\cdot}3' + 90 - \theta',$$

$$bB = 90° - \phi', \qquad s_2B = \rho', \qquad bs_2 = 40°\ 41',$$

and $\quad s_2bB = 89°\ 14{\cdot}6' - (\angle\beta - 90° + \lambda') = 62°\ 51' - \lambda'$.

Hence the relations in the spherical triangle bs_2B corresponding to (3) and (4) are:

$$\sin\phi' = \cos 40°\ 41' \cos\rho' - \sin 40°\ 41' \sin\rho' \sin(88°\ 7{\cdot}3' - \theta') \ldots\ldots(5)$$

and

$$\sin(62°\ 51' - \lambda') = \frac{\sin\rho' \cos(88°\ 7{\cdot}3' - \theta')}{\cos\phi'} \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

der Plagioklase. Chemie der Erde, Jena, 1915, vol. 1, p. 351; Inaug.-Diss., Heidelberg, 1916. [Min. Abstr., vol. 1, p. 390.]

[1] Throughout this paper, ρ, θ, ϕ, and λ for B are denoted as ρ', θ', ϕ', and λ' respectively, to avoid confusion with those for A.

The results of the calculation of ϕ' and λ' from the values of ρ' and θ' in Table VI by (5) and (6) are as follows:

TABLE VII.—Position of the Optic Axis B of Oligoclase. (Temperature 21° C.)

Wave-lengths (in $\mu\mu$).	ϕ'.	λ'.
700	+42° 33′	+61° 1′
644	+42 29	+61 11
589·3	+42 25	+61 22
535	+42 18	+61 38
508·5	+42 13	+61 44

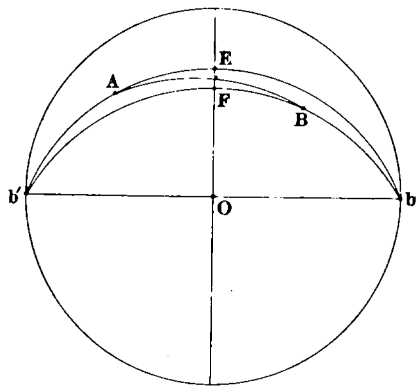

FIG. 5. Stereogram showing the relations between the positions of A and B and the optic axial angle; projected on the plane perpendicular to the c-axis. $EA = \phi(-)$, $OE = \lambda$, $FB = \phi'$, $OF = \lambda'$.

Optic Axial Angle.

The optic axial angle ($2V$) of the oligoclase was calculated in two different ways: (i) from the three principal refractive indices by the well-known relation

$$\tan\Omega = \frac{\gamma}{\alpha}\sqrt{\frac{(\beta+\alpha)(\beta-\alpha)}{(\gamma+\beta)(\gamma-\beta)}} \quad\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

where Ω denotes the angle between the optic elasticity axis Z and one of the optic binormals, and (ii) from ϕ, λ, ϕ', and λ' by the relation [1]

$$\cos AB = \sin\phi \sin\phi' + \cos\phi \cos\phi' \cos(\lambda-\lambda') \quad\ldots\ldots\ldots(8)$$

which is obtained from

$$\cos AB = \cos bA \cos bB + \sin bA \sin bB \cos AbB$$

in the spherical triangle bAB (fig. 5), by putting

$$bA = 90°-\phi, \qquad bB = 90°-\phi', \qquad AbB = \lambda-\lambda'.$$

[1] This relation holds good for any positions of A and B, taking signs of ϕ, λ, ϕ', and λ' into consideration.

The optic axial angles of the oligoclase calculated by (7) and (8) are given respectively in columns I and II of Table VIII.

In the present case, Ω is greater than $45°$, i.e. the mineral is optically negative for light of the wave-lengths $700-508{\cdot}5\ \mu\mu$, and

$$2V = 180° - 2\Omega = \grave{A}B.$$

TABLE VIII.—Optic Axial Angle of Oligoclase. (Temperature $20{\cdot}5°-21°$ C.)

Wave-lengths (in $\mu\mu$).	I.	II.
700	83° 45′	83° 28′
671	83 4	—
644	83 57	83 9
610	83 5	—
589.3	82 25	82 47
554	82 14	—
535	81 35	82 24
527	82 14	
508.5	81 22	82 11

I. Calculated from α, β, and γ.
II. Calculated from ϕ, λ, ϕ', and λ'.

II. ANDESINE.

The material examined was a small crystal of andesine from Maeyama, Shinano, Japan, 5 mm. in the longest diameter, bounded by $b(010)$, $c(001)$, $m(110)$, $M(1\bar{1}0)$, $f(130)$, $z(1\bar{3}0)$, and $y(\bar{2}01)$, flat parallel to b, and twinned according to the Carlsbad-law. This andesine is said to form phenocrysts in a dacitic rock, and its chemical composition is represented by $Or_7\ Ab_{56}\ An_{37}$ according to the analysis by T. Nishikawa.[1]

The optical constants for sodium-light of andesine from the same locality were determined by F. Becke and M. Goldschlag[2] with the results:

$$\alpha = 1{\cdot}5461 \qquad \beta = 1{\cdot}5498 \qquad \gamma = 1{\cdot}5531$$

$$A\begin{cases}\lambda = +80{\cdot}4°\\ \phi = -36{\cdot}3°\end{cases} \qquad B\begin{cases}\lambda' = +40{\cdot}0°\\ \phi' = +37{\cdot}1°\end{cases}$$

The material available to the present writer was not very suitable for making a complete study of the dispersion, owing to the small size of the crystal, its twinning and zonal structure; and so only the dispersion of the principal refractive indices was observed.

[1] T. Wada, Minerals of Japan, 1904, p. 135; revised edition (in Japanese), 1916, p. 184.

SiO_2.	Al_2O_3.	CaO.	Na_2O.	K_2O.	Total.
58·21	26·46	7·58	6·32	1·28	99·85

[2] F. Becke and M. Goldschlag, Sitzungsber. Akad. Wiss. Wien, Math.-naturw. Kl., 1918, vol. 127, Abt. I, p. 502; F. Becke, Tscherm. Min. Petr. Mitt., 1921, vol. 35, p. 31. [Min. Abstr., vol. 1, p. 391; vol. 2, p. 61.]

Refractive Indices and Birefringencies.

A polished plane surface practically parallel to $b(010)$ of the andesine was employed for determining refractive indices. On working with the refractometer it was found difficult at certain azimuths to observe accurately the boundary lines of total reflection, especially for the light near the blue end of the spectrum. The refractive indices of the andesine in the following table should therefore be regarded as approximate.

TABLE IX.—Refractive Indices of Andesine. (Temperature 22° C.)

Wave-lengths (in $\mu\mu$).	α.	β.	γ.
700	1·5408	1·5449	1·5486
671	1·5419	1·5459	1·5497
644	1·5427	1·5468	1·5506
610	1·5442	1·5482	1·5519
589·3	1·5450	1·5491	1·5529
554	1·5468	1·5508	1·5547
535	1·5478	1·5518	1·5559
527	1·5484	1·5523	1·5564
508·5	1·5496	1·5536	1·5574

The refractive indices for light of the wave-length 589·3 $\mu\mu$ in the above table are a little lower than those determined by F. Becke and M. Goldschlag. This may probably be accounted for as due largely to the difference of the materials examined.

Birefringencies calculated from the above values of the refractive indices are as follows:

TABLE X.—Birefringencies of Andesine. (Temperature 22° C.)

Wave-lengths (in $\mu\mu$).	$\gamma - \alpha$.	$\gamma - \beta$.	$\beta - \alpha$.
700	0·0078	0·0037	0·0041
671	0·0078	0·0038	0·0040
644	0·0079	0·0038	0·0041
610	0·0077	0·0037	0·0040
589·3	0·0079	0·0038	0·0041
554	0·0079	0·0039	0·0040
535	0·0081	0·0041	0·0040
527	0·0080	0·0041	0·0039
508·5	0·0078	0·0038	0·0040

III. LABRADORITE.

The material examined was a labradorite of high degree of homogeneity from County Down, Ireland. It is the same material as that described by A. Hutchinson and W. Campbell Smith.[1] According to them the specimen

[1] A. Hutchinson and W. Campbell Smith, Min. Mag., 1912, vol. 16, p. 264.

came from large phenocrystic constituents (8 by 5 cm. or larger) of the dykes (4 to 6 feet wide) occurring just below the lighthouse at St. John's Point, near Ardglass. The groundmass in which the phenocrysts are embedded is very compact, and consists largely of short laths of labradorite, the interstices between which are filled up by augite, magnetite, and green glass. The chemical composition of the felspar now under consideration is represented by $Or_{5\cdot0}\,Ab_{32\cdot7}\,An_{62\cdot3}$, according to the analyses by G. G. Knighton and M. C. Burkitt.[1]

The main optical data (for sodium-light) of the labradorite determined by A. Hutchinson and W. Campbell Smith are as follows:

$$\alpha = 1{\cdot}5630 \qquad \beta = 1{\cdot}5665 \qquad \gamma = 1{\cdot}5712,$$
$$2V = 81^\circ\,48' \qquad \text{Optically positive.}$$

Refractive Indices and Birefringencies.

The refractive indices of the labradorite were determined in the same way as in the case of the oligoclase.

TABLE XI.—Refractive Indices of Labradorite. (Temperature 21·5° C.)

Wave-lengths (in $\mu\mu$).	α.	β.	γ.
700	1·5577	1·5619	1·5668
671	1·5587	1·5629	1·5679
644	1·5596	1·5637	1·5686
610	1·5614	1·5654	1·5703
589·3	1·5623	1·5663	1·5713
554	1·5640	1·5680	1·5731
535	1·5652	1·5691	1·5742
527	1·5657	1·5696	1·5748
508·5	1·5671	1·5709	1·5761

TABLE XII.—Birefringencies of Labradorite. (Temperature 21·5° C.)

Wave-lengths (in $\mu\mu$).	$\gamma-\alpha$.	$\gamma-\beta$.	$\beta-\alpha$.
700	0·0091	0·0049	0·0042
671	0·0092	0·0050	0·0042
644	0·0090	0·0049	0·0041
610	0·0089	0·0049	0·0040
589·3	0·0090	0·0050	0·0040
554	0·0091	0·0051	0·0040
535	0·0090	0·0051	0·0039
527	0·0091	0·0052	0·0039
508·5	0·0090	0·0052	0·0038

[1] The analyses are given in the above cited paper (p. 270) by A. Hutchinson and W. Campbell Smith. The mean of the three analyses:

SiO_2.	Al_2O_3.	Fe_2O_3.	CaO.	Na_2O.	K_2O.	Ign. loss.	Total.
52·33	30·22	0·40	12·52	3·62	0·85	0·36	100·30

Positions of the Optic Axes.

The positions of the optic axes of the labradorite were determined with a section (s) orientated as shown in figs. 6 and 7, where

b (010) : c (001)	...	...	86° 8′
c (001) : m (110)	...	...	65 51
b (010) : m (110)	...	...	58 30
m (110) : s	..	...	62 20
c (001) : s	...	...	86 9

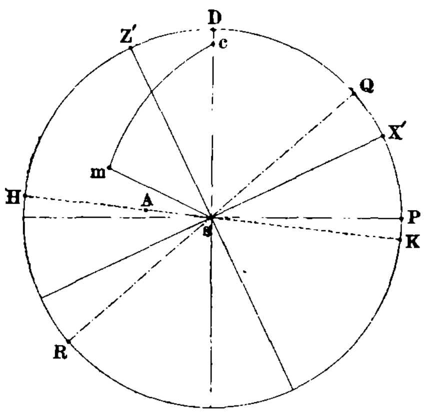

FIG. 6.

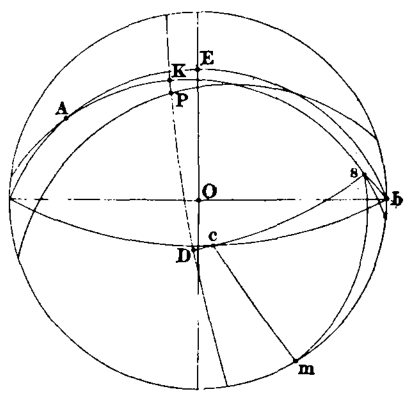

FIG. 7.

FIG. 6. Stereogram showing the positions of s and A of labradorite and also the directions of extinction (sX' and sZ') and of the trace of the optical plane (QR) in the section s; projected on the plane of s. $sA = \rho$, $PH = \theta$, $PX' = \epsilon$, $PQ = \delta$.

FIG. 7. Stereogram showing the positions of s and A of labradorite and the relations between ρ, θ, ϕ, and λ; projected on the plane perpendicular to the c-axis. A in this figure corresponds to the antipode of A in fig. 6. $sA = 180° - \rho$, $PK = 180° - \theta$, $EA = \phi\ (-)$, $OE = \lambda$.

From the above it follows that

$\angle \beta$	...	...	116° 4·4′
b (010) : s	...	...	4 14·8

The section did not show the emergence of the optic axes, A and B, in an ordinary conoscope, but the point of emergence of A was within the field of a large aperture conoscope provided with a $\frac{1}{12}$-inch objective immersed in cedarwood oil. With this conoscope, ρ and θ for A of the labradorite were measured in the same way as in the case of the oligoclase.

Since the section s was the only one whose orientation could be determined accurately, the writer tried to find ρ' and θ' for B also with this section. For this purpose the directions of extinction and of the trace of the optical plane in s were determined. They are represented in fig. 6, where sX' and sZ' are respectively the vibration-directions of the faster and slower waves in the section, and QR the trace of the optical plane. The results of the measurements are given, with ρ and θ, in Table XIII, where ϵ denotes the angle between sX' and sP (the trace of c), or PX', and δ the angle between QR and sP, or PQ.

For determining the extinction position a Bertrand quartz-plate ocular was used, and the angle was read at four azimuths in a complete rotation of the stage on which the section was put, the section being brought to extinction by rotating the stage first from one direction and then from the other. δ was determined also at four azimuths of the stage, such that the trace of the optical plane was parallel to one of the vibration-directions of the crossed nicols. Here, too, the section was brought to this position from both directions. Each reading of ϵ and δ was taken to 2·5′ by means of a vernier attached to the stage. The values given under ϵ and δ in Table XIII are the mean values obtained from repeated measurements.

TABLE XIII.

Wave-lengths (in $\mu\mu$).	ρ.	θ.	ϵ.	δ.
700	39° 58′	174° 34′	24° 27′	36° 51′
644	39 37	174 15	24 35	37 17
589·3	39 4	173 58	24 45	37 41
535	38 30	173 40	24 54	38 17
508·5	38 13	173 31	24 58	38 29

Position of the Optic Axis A.—By solving the spherical triangle bcs we obtain

$$bsc = 89° \, 37{\cdot}9' \quad \text{and} \quad cbs = 90° \, 4{\cdot}9' ;$$

we have also

$$csA = 270° - \theta,$$

$$bsA = 360° - bsc - csA = \theta + 0° \, 22{\cdot}1',$$

$$bA = 90° - \phi, \qquad sA = 180° - \rho, \qquad bs = 4° \, 14{\cdot}8',$$

and [1]

$$sbA = \lambda - (cbs - cbO) = \lambda - \{90° \, 4{\cdot}9' - (\angle\beta - 90°)\} = \lambda - 64° \, 0{\cdot}5'.$$

Hence we have in the spherical triangle bsA

$$\sin\phi = -\cos 4° \, 14{\cdot}8' \cos\rho + \sin 4° \, 14{\cdot}8' \sin\rho \cos(\theta + 0° \, 22{\cdot}1') \quad \ldots\ldots(9)$$

and

$$\sin(\lambda - 64° \, 0{\cdot}5') = \frac{\sin\rho \sin(\theta + 0° \, 22{\cdot}1')}{\cos\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

[1] $\angle\beta = 116° \, 4{\cdot}4'$ as was given before.

Table XIV gives ϕ and λ for A, calculated from ρ and θ in Table XIII by (9) and (10).

TABLE XIV.—Position of the Optic Axis A of Labradorite.
(Temperature 21·5° C.)

Wave-lengths (in $\mu\mu$).	ϕ.	λ.
700	−54° 16′	+69° 35′
644	−54 37	+69 56
589·3	−55 9	+70 16
535	−55 43	+70 36
508·5	−56 0	+70 47

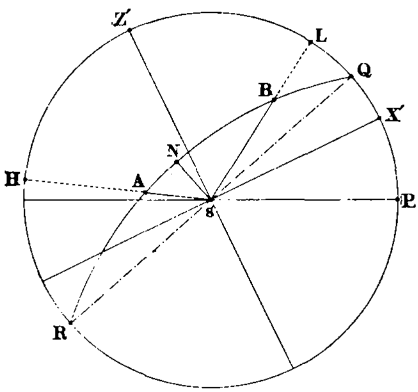

FIG. 8.

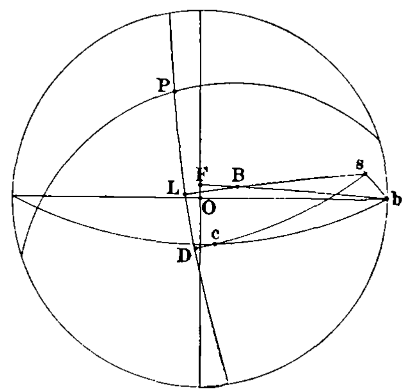

FIG. 9.

FIG. 8. Stereogram showing the relations between ρ, θ, ρ', θ', ϵ, and δ in the section s of labradorite; projected on the plane of s. $sA = \rho$, $PH = \theta$, $sB = \rho'$, $PL = \theta'$, $PX' = \epsilon$, $PQ = \delta$.

FIG. 9. Stereogram showing the relations between ρ', θ', ϕ', and λ' of labradorite; projected on the plane perpendicular to the c-axis. $sB = \rho'$, $PL = \theta'$, $FB = \phi'$, $OF = \lambda'$.

Position of the Optic Axis B.—The position of the optic axis B was found from the data in Table XIII in the following way:

In the stereogram of fig. 8, which was constructed in the same way as fig. 6, B must lie on the great circle QAR which represents the optical plane. B must also lie on the line sL, drawn so that the angle $Z'sL = Z'sH$ on the opposite side of sZ' with sH, by the Biot-Fresnel's law. The intersection of QAR and sL is therefore B; and sB and PL are respectively ρ' and θ' for B.

We have here, by construction,

$$\theta' = 180° - \theta + 2\epsilon \quad \text{..........................(11)}$$

In the spherical triangle sAN (N being the intersection of $QBAR$ with the perpendicular to QsR at s) we have

$$\tan sN = \tan \rho \cos AsN,$$

and in sBN
$$\tan \rho' = \frac{\tan sN}{\cos BsN};$$

while $AsN = (\theta - \delta) - 90°$ and $BsN = 90° - (\theta' - \delta)$;

therefore
$$\tan \rho' = \frac{\tan \rho \sin(\theta - \delta)}{\sin(\theta' - \delta)} \qquad \text{.......................(12)}$$

ρ' and θ' calculated by (11) and (12) are given in the table below.

TABLE XV.

Wave-lengths (in $\mu\mu$).	ρ'.	θ'.
700	61° 57′	54° 20′
644	61 48	54 55
589·3	61 21	55 32
535	61 15	56 8
508·5	61 8	56 25

As may be seen in fig. 9, we have

$$csB = 90° - \theta',$$
$$bsB = bsc + csB = 180° - (\theta' + 0°\, 22{\cdot}1'),$$

and
$$sbB = cbs - cbO - \lambda' = 64°\, 0{\cdot}5' - \lambda'.$$

Hence the relations for B corresponding to (9) and (10) for A are respectively:

$$\sin \phi' = \cos 4°\, 14{\cdot}8' \cos \rho' - \sin 4°\, 14{\cdot}8' \sin \rho' \cos(\theta' + 0°\, 22{\cdot}1') \text{...(13)}$$

and
$$\sin(64°\, 0{\cdot}5' - \lambda') = \frac{\sin \rho' \sin(\theta' + 0°\, 22{\cdot}1')}{\cos \phi'} \qquad \text{..............(14)}$$

The results of the calculation of ϕ' and λ' from ρ' and θ' in Table XV by the above relations are as follows:

TABLE XVI.—Position of the Optic Axis B of Labradorite. (Temperature 21·5° C.)

Wave-lengths (in $\mu\mu$).	ϕ'.	λ'.
700	+25° 33′	+11° 2′
644	+25 44	+10 29
589·3	+26 13	+ 9 55
535	+26 21	+ 9 20
508·5	+26 34	+ 9 4

Optic Axial Angle.

The optic axial angles of the labradorite for various wave-lengths were calculated from α, β, and γ by the relation (7), and also from ϕ, λ, ϕ', and

λ' by the relation (8). The results are as in Table XVII. Here Ω is less than $45°$, i. e. the mineral is optically positive for the light of the wave-lengths $700-508{\cdot}5\ \mu\mu$; and $2V = 2\Omega = 180° - AB$.

TABLE XVII.—Optic Axial Angle of Labradorite. (Temperature 21·5° C.)

Wave-lengths (in $\mu\mu$.)	I.	II.
700	85° 53′	85° 41′
671	85 16	—
644	85 9	84 54
610	84 26	—
589·3	83 52	83 45
554	83 18	—
535	82 35	82 52
527	82 2	—
508·5	81 18	82 19

I. Calculated from α, β, and γ.
II. Calculated from ϕ, λ, ϕ', and λ'.