An eyepiece micro-planimeter.

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OCCASION arose recently when the writer required to make a number of comparative areal measurements of irregularly shaped portions of fossil corals in thin section, and it was found that the micrometric methods available were very tedious and involved considerable strain. A micro-planimeter has now been designed by means of which fairly accurate determinations of areas can be made with a minimum of eye strain. Though intended primarily for use with the petrographical microscope with the rotating stage, it can be adapted for use on microscopes of the fixed stage type.

The device takes the form of a photographically reproduced micrometer scale which fits into the eyepiece of the microscope in place of the ordinary micrometer scale used for linear measurements. The zero point of the scale is in the centre of the field of view at the intersection of two cross lines which are at right angles. The scale is graduated in such a way that the graduations marked 1, 2, 3 . . . 10 are at distances from the zero point proportional to the radii of concentric circles of areas 1, 2, 3 . . . 10 units (fig. 1). If, therefore, a circular object is placed on the stage of the microscope with its centre at the zero point of the scale, the scale reading of the position of the circumference would be a measure of the area of the object, and the true area could be found by multiplying the scale reading by a factor dependent on the magnification of the particular lens combination in use. If the object is not circular but of irregular outline, and a very large number of readings of the position of the edge of the object were taken at points at equal angular distances apart through a complete rotation of 360°, the object might be considered to be subdivided into a large number of segments of circles of different areas (fig. 3*a*). If n such readings are taken, then each reading gives the area of a circle n times the area of the segment for which that reading was taken. If, therefore, the total of the n readings is T, the average value T/n is proportional to the true area of the object, and the true area can be found by multiplying by an appropriate factor. If an infinite number of readings is taken, the area so found must be the exact area, but it is found in practice that a close approximation can be arrived at with very few readings. For fairly regular objects twelve readings at angular intervals of 30° are usually sufficient, and with objects of extreme irregularity readings at 10° intervals give reliable results.

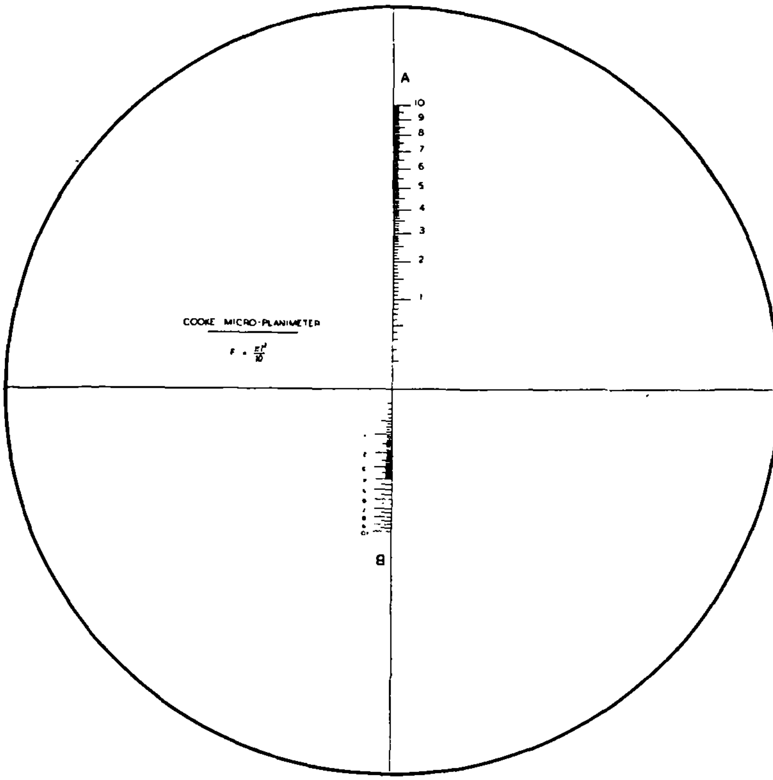

FIG. 1. The graduated scale comprising the micrometer.

The average of the readings taken is proportional to the area of the object, but to determine the true area the average must be multiplied by a factor depending on the lens combination in use. This factor can be readily determined by measuring the apparent length of the planimeter scale, from the centre to the graduation marked 10, by means of a stage micrometer. If this length is l mm., then a circular object of radius l mm. would give, with the objective used, a reading of 10 units for its area. Hence, since the true area is πl^2 sq. mm. and the apparent area 10 units, the true area is found by multiplying the reading by a factor F such that $F = \pi l^2/10$, and this factor is a constant for the particular objective (used in combination with the eyepiece in question). Fig. 2 plots F in terms of l. True areas of any object measured with the particular objective will therefore be found by multiplying the average value T/n by the factor F for the objective. It is also at once apparent

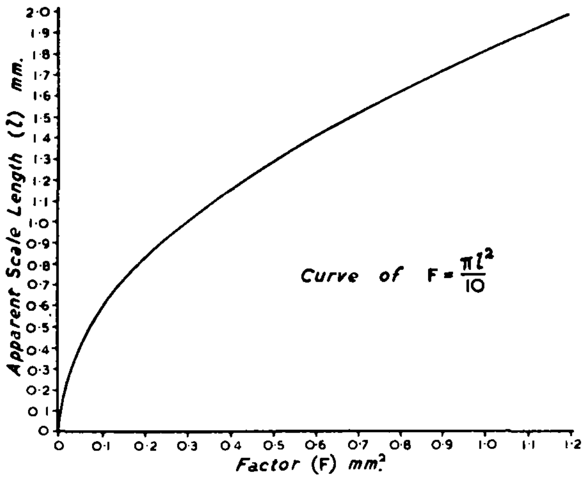

Fig. 2. Curve of the expression $F = \pi l^2/10$ for finding the value of the factor to be used in calculations with the micro-planimeter.

that a simpler factor ϕ can be used if n is kept constant. In this case the total of the readings T can be multiplied by ϕ, where $\phi = \pi l^2/10n$ or $\phi = F/n$, to give the same result, but *it must be remembered that* ϕ is dependent upon the value of n, and that F, and *not* ϕ, is the constant for the objective. From the formula, F may be determined for all the objectives in use, and this record should be kept with the microscope. It must, however, also be remembered that F takes into account the lower lens of the eyepiece in which the micrometer is mounted as well, and that a change of eyepiece may affect the factor. Since in practice only one eyepiece is generally fitted with a micrometer, this question does not commonly arise.

Two scales A and B have been incorporated in the micrometer, and are of exact known lengths, 5 mm. and 2·5 mm. respectively, though it has been found impossible for mechanical reasons to graduate B along the whole of its length. Two scales have been used because it was found that the first division of the larger scale was not sufficiently finely graduated to enable small and large objects in the same section to be measured without changing the objective and thus altering the factor. Since the factor for B is one-quarter that for A, a ready conversion is possible. Rotation of the eyepiece through 180° interchanges the positions of the scales A and B, which are so arranged that confusion in reading is not likely to arise. When the micrometer is used in some eyepieces it will be found that the whole of scale A may not be visible. This in no way affects its utility, but since the whole scale is not visible for measurement of its length, scale B can be measured and the factor for A can be taken as four times that for B.

(Experimental designs were tested, using a number of scales at various angular inclinations to one another, with the object of reducing the number of alterations which must be made in the angular position of the stage, but the simplest form seems best, since errors due to confusion are much less likely to occur.)

For use with microscopes with a fixed stage it is necessary to rotate the eyepiece relative to the object. A scale must be attached either to the microscope tube or to the eyepiece itself, and rotation effected against a register mark on the component to which the scale is not attached. Such a scale must be made to fit the particular microscope upon which it is to be used, and since the outer diameter of different makes of microscope tube differ so widely, it has not been possible to produce such a scale. It is a simple matter, however, to make one out of tin-foil, thick cardboard, or celluloid, and it need be graduated only every 10° with a heavier mark every 30°. A spot of white paint on the edge of the eyepiece rim, or a second attached ring on the tube with a register mark on its upper surface, enable quite accurate settings to be made. The attachment is used in the same manner as a cap analyser on petrographic microscopes of the 'Dick' type. A deviation of one degree makes very little difference to a reading, unless the edge of the object happens to be very oblique to the scale.

In using the micro-planimeter it is essential that the microscope should be accurately centred, since eccentricity on rotation produces additive or subtractive values on each reading with a total error quite disproportionate to the apparently small displacement at the centre. This disproportion will be seen to be accounted for by the fact that the segments into which the object is imagined to be divided are no longer in contact along their edges, but either overlap or are separated by a strip of rectangular shape with one side the length of the displacement of the centre from reading to reading. This is clearly shown in the diagram (fig. 3*b*) in which the eccentricity has been somewhat exaggerated.

Another precaution to be observed in taking readings is the avoidance of parallax errors due to the fact that the plane of the image does not always coincide with the plane in which the micrometer lies. If the eyepiece is fitted with a focusing eye-lens the focus can be adjusted to

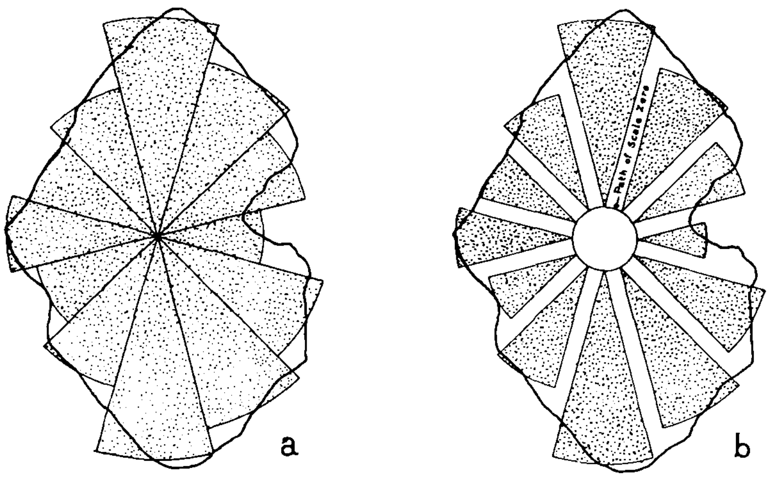

FIG. 3. Illustrating the error resulting from inaccurate centring of the objective. The stippled areas are the segments whose actual areas are measured. (*a*) is correctly centred and the actual error in the measured area is $+0{\cdot}34\%$. (*b*) is inaccurately centred and the error is obviously very large.

minimize this difference. It is advisable to use an objective of magnification such that the object to be measured fills about half of the field of view, and the centre of the object should be near the centre of the field. Before commencing readings, it is advisable to rotate the object completely to ensure that its outer edge does not at any point fall outside the limits of the scale.

Numerous experiments have been made to test the reliability of the micro-planimeter, both by measurements on regular objects of areas calculable from linear measurements (foraminifera, radiolaria, diatoms, well-formed crystals, and the like), and on irregular objects whose areas could be determined by other methods, such as the laborious one of 'counting squares' with a net ruling or of making determinations from camera lucida drawings both by ordinary planimetry or by the 'cutting out and weighing' method. As a result of the determinations it is possible to say that the error does not exceed $\pm 2\%$, provided that the number of readings taken is suitably adjusted to the degree of irregularity of the object.

It has been used with advantage wherever comparative areal or volumetric measurements are needed. Notable amongst these uses in mineralogy is its application to solving the problem of the composition of zoned felspars, since it is possible to use the instrument under crossed nicols if the stage is kept fixed and the eyepiece moved, or the nicols are rotated periodically so that the boundaries of the different zones are always distinct. It has also been used to find the relationship between included crystals and their hosts; to find the degree to which decomposition has advanced in crystals; to determine volumetrically the changes taking place in hydrothermal alteration of minerals; and other similar uses. In petrography it has been used both for making comparative measurements regarding coarseness of crystallization in different phases of an igneous intrusion and in obtaining modal analyses. In this latter case analyses accurate enough for the determination of the rock under Johannsen's[1] scheme of classification can be very quickly made, since each mineral grain can be measured in two to three minutes or even less. The method adopted has been to draw a rectangle of known area on the cover-glass of the section by means of a stencil and then to determine the area of each constituent mineral within the rectangle, or a mask with a rectangular aperture may be stuck directly on the cover-glass. The accuracy of the method has been proved by performing comparative tests with the Shand or Dollar integrator and the micro-planimeter, and the economy of the latter may enable it to be used where the expensive and heavy instrument is not available.

It may be noted that where comparative areas only are required, and only one objective is being used, it is necessary only to compare the values of T, provided n is also kept constant. If n is not the same for all the observations, T/n values can be compared, still without the necessity of taking F into account. If different objectives are used, however, the true areas only must be used.

The micrometer is now obtainable to order from Messrs. E. Leitz of Wetzlar, Germany, or through their agents, and will be known as the *Cooke micro-planimeter*. It has been made primarily to fit the eyepiece specially manufactured by that firm for the reception of micrometers of all types, but it will be possible to fit it in any make of eyepiece where the micro-

[1] A. Johannsen, A descriptive petrography of the igneous rocks. 1931, vol. 1, pp. 140–158.

meters are interchangeable. It has also been made of such a size that it can conveniently be used in eyepieces giving a wide field of view, and when it is to be used in microscopes having the narrow tube, it is strongly recommended that the Leitz periplanatic eyepiece for the reception of micrometers be used, as this eyepiece alone out of a large number which were tried enables the whole of both scales to be visible in the field. In eyepieces having a diaphragm less than one centimetre in diameter, the whole of scale A will not be visible, though this does not impair the utility of the device.

The writer wishes to express his indebtedness to Mr. D. R. Robinson for his helpful advice and co-operation in solving problems of design, and particularly for his kindness and skill in drafting the originals from which the experimental and final micrometers were reproduced. He wishes also to thank Dr. F. Coles Phillips for his interest and assistance in preparing and presenting this paper.