## The structure of wadeite.

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[Taken as read January 27, 1956.]

## Introduction.

WADEITE has been found only in a suite of leucite-lamproites from the West Kimberley area of Western Australia. The leucite-bearing rocks occur at a number of isolated volcanic vents that have in every instance broken through sedimentary deposits of Permian age. ${ }^{1}$ The nature of the mineral, wadeite, has been described by Prider. ${ }^{2}$ It is a potassium-zirconium silicate occurring as highly birefringent prisms up to 1 mm . in length with hexagonal-shaped basal sections. It is invariably replaced in part along the poor pyramidal cleavages by calcite. These cleavages, intersecting at angles of nearly $90^{\circ}$, are rendered visible because of the presence of the secondary calcite which has entered along these directions. The mineral is colourless, uniaxial, optically positive, and has $\epsilon 1.655, \omega 1.625$, and $\epsilon-\omega 0 \cdot 030$. It has a specific gravity of $3 \cdot 10$, is non-magnetic, and insoluble in hot acids. Prider, on the basis of a chemical analysis (table I), calculated the formula to be

$$
\mathrm{K}_{2 \cdot 05}(\mathrm{Ca}, \mathrm{Na})_{0 \cdot 95}(\mathrm{Zr}, \mathrm{Ti}, \mathrm{Al})_{1 \cdot 23}(\mathrm{Si}, \mathrm{P}, \mathrm{Al})_{4 \cdot 00} \mathrm{O}_{12}
$$

X-ray powder and Laue diffraction work, carried out in this department by Shearer, Robinson, and Jefferies, ${ }^{3}$ showed the diffraction symmetry to be $6 / m\left(C_{6 h}\right)$ and the unit-cell dimensions to be $a 6.89$ and $c 10 \cdot 18 \AA$. On the basis of this information and the chemical data, the formula ascribed to wadeite was $\left(\mathrm{K}(\mathrm{Zr}, \mathrm{Ca}) \mathrm{Si}_{2} \mathrm{O}_{6}\right)_{3}$. The purpose of the present work was to confirm these earlier observations and to attempt a complete structure determination of what appeared to be a unique mineral structure.
${ }^{1}$ A. Wade and R. T. Prider, Quart. Journ. Geol. Soc. London, 1940, vol. 96, p. 39. [M.A. 8-28.]
${ }^{2}$ R. T. Prider, Min. Mag., 1939, vol. 25, p. 379.
${ }^{3}$ J. Shearer, E. T. Robinson, and J. T. Jefferies, Min. Mag., 1952, vol. 29, Proc. p. lxi.

Table I. Chemical analysis of wadeite.

|  | Molecular proportions (Prider). | No. of metal atoms on basis of $18(0)$. | Valency. | Ionic radii (Å.). | Grouping. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}_{2} \mathrm{O}_{5}$ | 0.022 | $0 \cdot 34$ | 5 | $0 \cdot 34$ |  |
| $\mathrm{SiO}_{2}$ | 0.657 | $5 \cdot 06$ | 4 | 0.39 | $6(\mathrm{Si}, \mathrm{Al}, \mathrm{P})^{4+}$ |
| $\mathrm{Al}_{2} \mathrm{O}_{3}$ | 0.059 | 0.91 | 3 | 0.57 |  |
| MgO | 0.007 | $0 \cdot 06$ | 2 | $0 \cdot 78$ |  |
| $\mathrm{TiO}_{2}$ | 0.020 | $0 \cdot 15$ | 4 | $0 \cdot 64$ | 2( $\mathrm{Zr}, \mathrm{Ti}, \mathrm{Mg}, \mathrm{Al}, \mathrm{Na})^{4+}$ |
| $\mathrm{ZrO}_{2}$ | $0 \cdot 173$ | $1 \cdot 33$ | 4 | 0.87 |  |
| $\mathrm{Na}_{2} \mathrm{O}$ | 0.045 | $0 \cdot 69$ | 1 | 0.98 |  |
| CaO | 0.093 | 0.73 | 2 | $1 \cdot 06$ |  |
| $\mathrm{K}_{2} \mathrm{O}$ | $0 \cdot 196$ | $3 \cdot 02$ | 1 | 1.33 | 4(K, $\mathrm{Ca}, \mathrm{Ba}, \mathrm{Na})^{+}$ |
| BaO | $0 \cdot 008$ | $0 \cdot 06$ | 2 | $1 \cdot 43$ |  |
|  |  | 12.35 |  |  |  |
| Oxygen |  | 18 | -2 | $1 \cdot 32$ | $180^{2-}$ |

## Expertmental.

The crystal used was of the shape and size shown in fig. 1. The direction of the $c$-axis was reasonably well defined but there was only one $120^{\circ}$ interfacial angle that could be distinguished.


Fig. 1. Size of the crystal used and general appearance of the cleavage cracks.
Single-crystal oscillation and Weissenberg diffraction patterns were prepared using copper and molybdenum $K \alpha$ radiations. The unit-cell dimensions were determined by the $\theta$-method ${ }^{1}$ to be $a 6.893 \pm 0.004 \AA$. and $c 10 \cdot 172 \pm 0 \cdot 002 \AA$. The intensities of the X-ray reflections were estimated visually on a series of Weissenberg photographs and the appropriate absorption, Lorentz, and polarization corrections were applied. The observed structure amplitudes ( $F_{0}$ ) so obtained are listed in tables II and III. In order to check the accuracy of the visual estimation of intensity, a number of reflection intensities were measured on a Geiger-counter spectrometer. As can be seen from table IV, the agree-

[^0]ment is quite reasonable, except for some of the weaker reflections and the strong reflection ( $2 \overline{11} 2$ ). Inaccuracy in the visual method is to be expected for such very strong and weak reflections. ${ }^{1}$

Table II. Observed and calculated structure amplitudes $\mathrm{F}(h k i 0)$.

| $h$ | $k$ | $i$ | $l$. | $F_{0}$. | $F_{c}$. | $h$ | $k$ | $i$ | $l$. | $F_{0}$. | $F_{C}{ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | - | 356 | 5 | 3 | $\overline{8}$ | 0 | 35 | 36 |
| 1 | 0 | $\overline{1}$ | 0 | - | 41 | 6 | 2 | $\overline{8}$ | 0 | 10 | 13 |
| 2 | $\overline{1}$ | $\overline{1}$ | 0 | 68 | 68 | 7 | 1 | $\overline{8}$ | 0 | 37 | 37 |
| 2 | 0 | $\overline{2}$ | 0 | $<8$ | 7 | 8 | 0 | $\overline{8}$ | 0 | 50 | 46 |
| 3 | $\overline{1}$ | $\overline{2}$ | 0 | 42 | 48 | 9 | $\overline{1}$ | $\overline{8}$ | 0 | 22 | 18 |
| 4 | $\overline{2}$ | $\overline{2}$ | 0 | 81 | 86 | 10 | $\overline{2}$ | $\overline{8}$ | 0 | 23 | 29 |
| 2 | 1 | $\overline{3}$ | 0 | $<5$ | 7 | 11 | $\overline{3}$ | $\overline{8}$ | 0 | $<\mathrm{II}$ | 1 |
| 3 | 0 | $\overline{3}$ | 0 | 95 | 95 | 12 | $\overline{4}$ | $\overline{8}$ | 0 | $<11$ | 8 |
| 4 | $\overline{1}$ | $\overline{3}$ | 0 | 75 | 80 | 13 | $\overline{5}$ | $\overline{8}$ | 0 | $<11$ | 8 |
| 5 | $\overline{2}$ | $\overline{3}$ | 0 | 8 | 9 | 5 | 4 | $\overline{9}$ | 0 | 40 | 32 |
| 6 | $\overline{3}$ | $\overline{3}$ | 0 | 46 | 51 | 6 | 3 | $\overline{9}$ | 0 | 56 | 60 |
| 3 | 1 | $\overline{4}$ | 0 | $<6$ | 3 | 7 | 2 | $\overline{9}$ | 0 | $<9$ | 1 |
| 4 | 0 | $\overline{4}$ | 0 | 38 | 40 | 8 | 1 | $\overline{9}$ | 0 | $<10$ | 3 |
| 5 | 1 | 4 | 0 | 65 | 67 | 9 | 0 | $\overline{9}$ | 0 | 27 | 28 |
| 6 | $\overline{2}$ | $\overline{4}$ | 0 | 31 | 28 | 10 | I | $\overline{9}$ | 0 | $<11$ | 5 |
| 7 | $\overline{3}$ | $\overline{4}$ | 0 | 37 | 36 | 11 | $\overline{2}$ | $\overline{9}$ | 0 | $<11$ | 5 |
| 8 | $\overline{4}$ | $\overline{4}$ | 0 | 48 | 50 | 12 | $\overline{3}$ | $\overline{9}$ | 0 | $<11$ | 14 |
| 3 | 2 | 5 | 0 | 13 | $\overline{12}$ | 13 | $\overline{4}$ | $\overline{9}$ | 0 | 22 | 11 |
| 4 | 1 | $\overline{5}$ | 0 | 106 | 111 | 6 | 4 | $\overline{1} 0$ | 0 | $<10$ | 4 |
| 5 | 0 | $\overline{5}$ | 0 | 33 | 27 | 7 | 3 | $\overline{1} 0$ | 0 | $<10$ | 0 |
| 6 | $\overline{1}$ | $\overline{5}$ | 0 | 25 | 19 | 8 | 2 | $\overline{10}$ | 0 | 28 | 34 |
| 7 | $\overline{2}$ | $\overline{5}$ | 0 | 60 | 60 | 9 | 1 | $\overline{1} 0$ | 0 | $<11$ | 3 |
| 8 | $\overline{3}$ | $\overline{5}$ | 0 | $<8$ | 7 | 10 | 0 | $\overline{10}$ | 0 | $<11$ | 5 |
| 9 | $\overline{4}$ | $\overline{5}$ | 0 | $<9$ | 1 | 11 | $\overline{\mathrm{I}}$ | $\overline{1} 0$ | 0 | 30 | 29 |
| 10 | $\overline{5}$ | $\overline{5}$ | 0 | 18 | 24 | 12 | $\overline{2}$ | 10 | 0 | $<12$ | 11 |
| 4 | 2 | $\overline{6}$ | 0 | 21 | 14 | 13 | $\overline{3}$ | 10 | 0 | $<12$ | 0 |
| 5 | 1 | $\overline{6}$ | 0 | 33 | 23 | 6 | 5 | $\overline{1}$ | 0 | $<11$ | 2 |
| 6 | 0 | $\overline{6}$ | 0 | 33 | 37 | 7 | 4 | 11. | 0 | 16 | 26 |
| 7 | $\overline{1}$ | $\overline{6}$ | 0 | $<8$ | 4 | 8 | 3 | 11. | 0 | 15 | 12 |
| 8 | $\overline{2}$ | $\overline{6}$ | 0 | $<8$ | 2 | 9 | 2 | $\overline{11}$ | 0 | 21 | 23 |
| 9 | $\overline{3}$ | $\overline{6}$ | 0 | 42 | 45 | 10 | 1 | $\overline{11}$ | 0 | $<11$ | 18 |
| 10 | $\overline{4}$ | $\overline{6}$ | 0 | 18 | 14 | 11 | 0 | 11 | 0 | $<11$ | 1 |
| 11 | $\overline{5}$ | $\overline{6}$ | 0 | 15 | 12 | 12 | $\overline{1}$ | Ī | 0 | $<12$ | 9 |
| 12 | $\overline{6}$ | $\overline{6}$ | 0 | 32 | 29 | 7 | 5 | $\overline{1} \overline{2}$ | 0 | 18 | 12 |
| 4 | 3 | $\overline{7}$ | 0 | 16 | 17 | 8 | 4 | $\overline{1} 2$ | 0 | < 11 | 6 |
| 5 | 2 | $\overline{7}$ | 0 | 39 | 51 | 9 | 3 | 12 | 0 | 25 | 30 |
| 6 | 1 | $\overline{7}$ | 0 | $<8$ | 3 | 10 | 2 | 12 | 0 | $<12$ | 9 |
| 7 | 0 | $\overline{7}$ | 0 | 23 | 22 | 11 | 1 | $\overline{1} 2$ | 0 | $<12$ | 0 |
| 8 | $\overline{1}$ | $\overline{7}$ | 0 | 51 | 65 | 12 | 0 | $\overline{12}$ | 0 | 15 | 18 |
| 9 | $\stackrel{\rightharpoonup}{2}$ | $\overline{7}$ | 0 | $<9$ | 8 | 7 | 6 | $\overline{13}$ | 0 | $<12$ | 0 |
| 10 | $\overline{3}$ | $\overline{7}$ | 0 | 23 | 15 | 8 | 5 | $\overline{13}$ | 0 | $<12$ | 13 |
| 11 | $\overline{4}$ | $\overline{7}$ | 0 | 28 | 32 | 9 | 4 | $\overline{1} \overline{3}$ | 0 | $<12$ | 4 |
| 12 | $\overline{5}$ | $\overline{7}$ | 0 | $<11$ | 10 | 10 | 3 | $\overline{13}$ | 0 | $<12$ | 1 |
| 13 | $\overline{6}$ | $\overline{7}$ | 0 | $<12$ | 6 |  |  |  |  |  |  |

Table III. Observed and calculated structure amplitudes $\mathbf{F}(2 h \bar{h} \bar{h} l)$.
The letter $d$ indicates that the reflection conld be detected but not measured.

$$
R=\left(F_{c}-F_{0}\right) / F_{c}
$$

| $h$ | $l$. | $F_{0}$. | $\boldsymbol{F}_{\boldsymbol{c}}$. | $R$. | $h$ | $l$. | $F_{0}$. | $\boldsymbol{F}_{\boldsymbol{c}}$. | $R$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | - | 356 | - | 4 | 15 | - | $\overline{1}$ | - |
| 0 | 2 | - | 9 | - | 4 | 16 | - | 26 | - |
| 0 | 4 | 77 | 82 | 0.06 | 4 | 17 | - | 1 | - |
| 0 | 6 | 61 | $\overline{6} 4$ | 0.05 | 4 | 18 | - | 16 | - |
| 0 | 8 | 91 | 87 | 0.05 | 4 | 19 | -- | $\overline{\mathbf{I}}$ | - |
| 0 | 10 | d | $\overline{2} \overline{0}$ | - | 6 | 0 | 47 | 52 | $0 \cdot 10$ |
| 0 | 12 | 45 | 41 | $0 \cdot 10$ | 6 | 1 | - | $\overline{3}$ | - |
| 0 | 14 | - | $\overline{8}$ | - | 6 | 2 | 64 | 58 | $0 \cdot 10$ |
| 0 | 16 | 55 | 54 | $0 \cdot 02$ | 6 | 3 | 23 | 20 | $0 \cdot 15$ |
| 0 | 18 | d | 18 | - | 6 | 4 | 33 | 39 | $0 \cdot 15$ |
| 0 | 20 | d | 21 | - | 6 | 5 | d | 15 | - |
| 2 | 0 | 63 | 68 | 0.07 | 6 | 6 | 32 | 22 | $0 \cdot 45$ |
| 2 | 1 | 50 | $\overline{43}$ | $0 \cdot 16$ | 6 | 7 | - | 0 | - |
| 2 | 2 | 103 | 110 | 0.06 | 6 | 8 | - | $\overline{12}$ | - |
| 2 | 3 | 19 | $\overline{19}$ | 0.00 | 6 | 9 | - | $\overline{6}$ | - |
| 2 | 4 | 59 | 54 | $0 \cdot 09$ | 6 | 10 | -- | 5 | - |
| 2 | 5 | - | $\overline{2}$ | - | 6 | 11 | - | 12 | - |
| 2 | 6 | 50 | 41 | $0 \cdot 22$ | 6 | 12 | - | 14 | - |
| 2 | 7 | 35 | 32 | 0.09 | 6 | 13 | - | 4 | - |
| 2 | 8 | - | 13 | - | 6 | 14 | - | 24 | - |
| 2 | 9 | d | 14 | - | 6 | 15 | - | $\stackrel{\rightharpoonup}{1}$ | - |
| 2 | 10 | d | 14 | - | 6 | 16 | - | 18 | - |
| 2 | 11 | - | $\overline{3}$ | - | 6 | 17 | - | $\overline{4}$ | - |
| 2 | 12 | - | 14 | 一 | 6 | 18 | - | 14 | - |
| 2 | 13 | - | $\overline{8}$ | - | 8 | 0 | 51 | 50 | $0 \cdot 02$ |
| 2 | 14 | - | 37 | - | 8 | 1 | - | $\overline{3}$ | - |
| 2 | 15 | - | 12 | - | 8 | 2 | 59 | 57 | $0 \cdot 04$ |
| 2 | 16 | - | 23 | - | 8 | 3 | - | 3 | - |
| 2 | 17 | - | $\overline{1}$ | - | 8 | 4 | - | 5 | - |
| 2 | 18 | - | 22 | - | 8 | 5 | - | $\overline{3}$ | - |
| 2 | 19 | - | $\overline{2}$ | - | 8 | 6 | d | 18 | - |
| 2 | 20 | - | 13 | - | 8 | 7 | - | 2 | - |
| 4 | 0 | 78 | 86 | 0.09 | 8 | 8 | - | $\overline{4}$ | - |
| 4 | 1 | - | 4 | - | 8 | 9 | - | $\overline{2}$ | - |
| 4 | 2 | 58 | 63 | 0.08 | 8 | 10 | - | 16 | - |
| 4 | 3 | - | $\overline{4}$ | - | 8 | 11 | - | 1 | - |
| 4 | 4 | 65 | 59 | $0 \cdot 10$ | 8 | 12 | - | 1 | - |
| 4 | 5 | - | 3 | - | 8 | 13 | - | 0 | - |
| 4 | 6 | d | 13 | - | 8 | 14 | - | 15 | - |
| 4 | 7 | - | $\overline{2}$ | - | 8 | 15 | - | 0 | - |
| 4 | 8 | - | 9 | - | 8 | 16 | - | 14 | - |
| 4 | 9 | - | 1 | - | 10 | 0 | 22 | 24 | $0 \cdot 08$ |
| 4 | 10 | - | 6 | - | 10 | 1 | - | 11 | - |
| 4 | 11 | - | $\bar{\square}$ | - | 10 | 2 | 35 | 41 | $0 \cdot 15$ |
| 4 | 12 | - | 24 | - | 10 | 3 | - | $\overline{16}$ | - |
| 4 | 13 | - | 1 | - | 10 | 4 | - | 13 | - |
| 4 | 14 | - | 24 | - | 10 | 5 | -- | 12 | - |

## Table III (contd.)

| $h$ | $l$. | $F_{0}$. | $F_{c}$. | $R$. | $h$ | $l$. | $\boldsymbol{F}_{0}$. | $F_{c} \cdot$ | $R$. |
| ---: | ---: | ---: | ---: | :--- | :---: | ---: | :--- | ---: | :--- |
| 10 | 6 | - | 17 | - | 12 | 1 | - | $\overline{5}$ | - |
| 10 | 7 | - | $\overline{7}$ | - | 12 | 2 | - | 21 | - |
| 10 | 8 | - | 8 | - | 12 | 3 | - | 5 | - |
| 10 | 9 | - | 8 | - | 12 | 4 | - | 12 | - |
| 10 | 10 | - | 6 | - | 12 | 5 | - | $\overline{4}$ | - |
| 10 | 11 | - | $\overline{8}$ | - | 12 | 6 | - | 2 | - |
| 10 | 12 | - | 4 | - | 12 | 7 | - | 4 | - |
| 10 | 13 | - | 5 | - | 12 | 8 | - | 20 | - |
| 10 | 14 | - | 16 | - | 12 | 9 | - | $\overline{4}$ | - |
| 12 | 0 | 31 | 29 | 0.07 | 12 | 10 | - | 1 | - |

Table IV. Comparison of counter intensities ( $I_{c}$ ) with visual intensities $\left(I_{v}\right)$.

| $h$ | $k$ | $i$ | $l$. | $I_{c}$. | $I_{v}$. | $h$ | $k$ | $i$ | $l$. | $I_{c}$. |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | $\overline{2}$ | $\overline{2}$ | 0 | 51 | 51 | 0 | 0 | 0 | 8 | 36 |
| 6 | $\overline{3}$ | $\overline{3}$ | 0 | 12 | 10 | 0 | 0 | 0 | 10 | 6 |
| 8 | $\overline{4}$ | $\overline{4}$ | 0 | 10 | 8 | 0 | 0 | 0 | 12 | 6 |
| 10 | $\overline{5}$ | $\overline{5}$ | 0 | $1 \cdot 2$ | $0 \cdot 8$ | 0 | 0 | 0 | 16 | 6 |
| 12 | $\overline{6}$ | $\overline{6}$ | 0 | $1 \cdot 8$ | $1 \cdot 9$ | 2 | $\overline{1}$ | $\overline{1}$ | 2 | 152 |
| 0 | 0 | 0 | 4 | 50 | 50 | 4 | $\overline{2}$ | $\frac{2}{2}$ | 4 | 28 |
| 0 | 0 | 0 | 6 | 21 | 21 | 6 | 3 | 3 | 6 | 4 |

An interesting feature of the oscillation photographs was the occurrence of a number of powder diffraction lines superimposed on the single crystal pattern. Reference to the American Society for Testing Materials index showed that the spacings of these lines corresponded to those of calcite, thus confirming the presence of this mineral in the surface cracks of the crystal.

## Solution of structure.

Examination of the zero-level and first-level Weissenberg photographs of the crystal taken with the $c$-axis and the ( $2 \overline{1} \overline{1} 0$ )-axis (perpendicular to the $a$ - and $c$-axes) as the axes of rotation, confirmed the diffraction symmetry of $6 / m\left(C_{6 h}\right)$. This, together with the systematically absent reflections, $000 l$ when $l$ is odd, indicated that wadeite belongs either to the space group $P 6_{3}\left(C_{6}^{6}\right)$ or $P 6_{3} / m\left(C_{6 h}^{2}\right)$.

Consideration of the unit-cell dimensions and the measured density shows there is one molecule, containing eighteen oxygen atoms, per unit cell (table I). Furthermore, general considerations of symmetry and chemical data, together with the failure of tests for non-centrosymmetry, suggest the formula to be $\mathrm{Zr}_{2} \mathrm{~K}_{4} \mathrm{Si}_{6} \mathrm{O}_{18}$ and that it belongs to the $P 6_{3} / m\left(C_{6 h}^{2}\right)$ space group. It is obvious from table I that this is only an idealized formula, since there has been considerable substitution of the metal atoms.

It has been noted that the cleavage cracks in the prism faces of the
crystal cross at nearly $90^{\circ}$. This is a typical property of pyroxenes which have, in general, a somewhat similar type of chemical formula to wadeite. However, it is impossible to fit a pyroxene type of siliconoxygen chain into the symmetry of the $P 6_{3} / m$ space group.

Wadeite, then, appears to be based on a silicon-oxygen framework of six silicon and eighteen oxygen atoms in the unit cell. By constructing a model of wooden spheres,


Fig. 2. Form of the temperature factor plotted against $(\sin \theta) / \lambda$. representing oxygen atoms, within the confines of a unit cell of appropriate dimensions, it was found that there were two, not very different, frameworks that fulfilled the required symmetry conditions. The structure based on either of these two frameworks defined the $x, y$ coordinates of the zirconium and potassium atoms; i.e. $(0,0, z)$ for zirconium and $(2 / 3,1 / 3, z)$ and $(1 / 3,2 / 3, z)$ for potassium.

Two heavy atoms, such as zirconium, at the origin of the twodimensional projection along the $c$-axis, provide a large, positive contribution to the structure factors corresponding to reflections of the type ( $h \mathrm{k} i 0$ ). There would therefore be very few errors in assuming all these structure factors to be positive. A Fourier synthesis of this projection was carried out using the values of $F_{0}$ in table II and taking them all to be positive. This synthesis showed the positions of all the atoms except for six oxygens, and established which of the two model structures was to be accepted.

In order to calculate the structure factors from atomic co-ordinates, it is necessary to know the atomic scattering factors for the atoms involved. For silicon and oxygen, these are listed in available tables. However, reference to the chemical data in table I shows that the nature of the other atoms in a unit cell is varied. A weighted mean of the atomic weights of the elements likely to be in the 'potassium' position corresponded closely to that of potassium, whose $f_{0}$ values are given directly in the tables. A similar mean for the 'zirconium' position suggested a quadruply-ionized element of atomic number 36. It was therefore necessary to construct an atomic scattering factor curve appropriate to this average atomic number. The values given in table V have been derived from those of bromine (35) and rubidium (37).

Table V. Atomic scattering factor $f_{0}$ for ' $\mathrm{Zr}^{4+}$ '.

| $(\sin \theta) / \lambda$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $f_{0}$ | $\cdots$ | 32.0 | 31.2 | 28.4 | 24.5 | 21.1 | 18.7 | 16.6 | 14.6 | 12.8 | 11.2 | 9.9 |

Refinements were made to the atomic parameters given by the model, using trial and error methods, until the best agreement was obtained


Frg. 3. Projection of electron density along the $c$-axis of wadeite. The contours are at the arbitrary levels of $0,1,2,3,4,5,10$, except at the three zirconium positions where they are at 0,10 , and 20 .
between observed and calculated values of the structure factors. The original values of the calculated structure factors had to be multiplied by a factor to reduce their numerical values to the arbitrary scale of the observed structure factors. This factor consists of a constant times the temperature factor. The experimentally determined curve (fig. 2), which includes a small absorption component, corresponds reasonably well to an exponential temperature factor of the form $e^{-B \sin ^{2} \theta}$, where $B=0.35$. Final values of $F_{c}$ are listed in table II.

A two-dimensional Fourier synthesis was carried out using reflections of the form ( $h k i 0$ ) ; the result is shown in fig. 3. All values of $F_{0}$ up to $\sin \theta / \lambda=1$ were used and where these were not observed, owing
to their low intensity, calculated values were substituted. The projection substantially confirms $x$ - and $y$-co-ordinates listed in table VI, although it suggests $(0 \cdot 387,0 \cdot 256, z)$ and $(0 \cdot 258,0 \cdot 236, z)$ might be more appropriate for the silicon position and the general oxygen position respectively. Adoption of these values does not, however, improve the agreement of $F_{0}$ and $F_{c}$.

For the second set of reflections investigated ( $2 h \bar{h} \bar{h} l$ ), only 19 significant observed structure factors could be measured out of a total of 108. This is somewhat unsatisfactory and the usefulness of a Fourier projection would be very doubtful. Nevertheless, the agreement obtained, as indicated by $R=\left(F_{c}-F_{0}\right) / F_{c}$ in table III, is sufficient to verify the structure already postulated. There is one serious difference between $F_{0}$ and $F_{0}$ for ( $6 \overline{3} \overline{3} 6$ ), but this is a relatively weak reflection and thus liable to a large error in measurement. It must be noted that the calculated structure factors for the ( $2 h \bar{h} \bar{h} l)$ reflections as listed in table III have involved the use of the correlating factor derived experimentally for the ( $h k i 0$ ) reflections. This is strictly not justified, since the temperature factor will not be the same in all directions in the wadeite crystal, but the error will not affect the results. The final values for all the atomic co-ordinates are tabulated in table VI.

Table VI. Co-ordinates of the atoms of wadeite expressed as fractions of $a, a$ and $c$.

| 120 | (0.263 | 0.236 | 0.114) | (0.764 | 0.027 | 0.114) | (0.973 | 0.737 | 0.114) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (0.737 | 0.764 | 0.886) | (0.236 | 0.973 | 0.886) | (0.027 | 0.263 | 0.886) |
|  | (0.737 | 0.764 | 0.664) | (0.236 | 0.973 | 0.664) | (0.027 | $0 \cdot 263$ | 0.664) |
|  | (0.263 | $0 \cdot 236$ | 0.386) | (0.764 | 0.027 | 0.386) | (0.973 | 0.737 | 0.386) |
| 60 | (0.500 | $0 \cdot 100$ | 0.250) | (0.900 | $0 \cdot 400$ | 0.250) | (0.600 | 0.500 | 0.250) |
|  | (0.500 | 0.900 | 0.750) | (0.100 | $0 \cdot 600$ | 0.750) | (0.400 | 0.500 | 0.750) |
| 6Si | (0.389 | 0.255 | $0 \cdot 250)$ | (0.745 | $0 \cdot 134$ | 0.250) | (0.866 | 0.611 | 0.250) |
|  | (0.611 | $0 \cdot 745$ | $0 \cdot 750$ ) | (0.255 | 0.866 | 0.750 ) | (0.134 | $0 \cdot 389$ | 0.750) |
| 4K | (0.666 | $0 \cdot 333$ | 0.558) | (0.666 | 0.333 | 0.942) | - | - | - |
|  | (0.333 | $0 \cdot 666$ | 0.442) | (0.333 | $0 \cdot 666$ | 0.058) | - | - | - |
| 2 Zr | $\left(\begin{array}{ll}0 & 0\end{array}\right.$ | 0) | (0) 0 | 500) |  |  |  |  |  |

## Description of structure.

The structure of wadeite is based on a framework of silica tetrahedra, the potassium and zirconium atoms fitting into the interstices. The fundamental unit of this framework is the $\left(\mathrm{Si}_{3} \mathrm{O}_{9}\right)^{6-}$ group (fig. 4), two of which combine with $4 \mathrm{~K}^{+}$and $2 \mathrm{Zr}^{4+}$ to form the neutral contents of


Fig. 4. The $\left(\mathrm{Si}_{3} \mathrm{O}_{9}\right)^{6-}$ unit, which consists of three silicon-oxygen tetrahedra forming a ring.

Fig. 5. Arrangement of the metal atoms within the silicon-oxygen framework. The oxygen atoms are at the corners of the figure.


Fig. 6. Grouping of the $\left(\mathrm{Si}_{3} \mathrm{O}_{9}\right)^{6-}$ units in the unit cell and the symmetry elements of the $P 6_{3} / m$ space group. The shaded groups are separated from the unshaded by a screw component of $c / 2$.
the unit cell. The $\left(\mathrm{Si}_{3} \mathrm{O}_{9}\right)^{6-}$ groups are arranged about the hexad screw axis as illustrated in fig. 5 .

The oxygen atoms are grouped in the manner indicated in fig. 5, where an oxygen atom is situated at each corner of the figure. The
zirconium atom is then situated at the centre of the octahedron formed by the six oxygen atoms; i.e. at the positions $(0,0,0)$ and ( $0,0, \frac{1}{2}$ ). Mirror reflection planes intersect the hexad screw axis $\left(6_{3}\right)$ at $z=\frac{1}{4}$ and $z=\frac{3}{4}$, perpendicularly between the octahedra. The $\left(\mathrm{Si}_{3} \mathrm{O}_{9}\right)^{6-}$ groups lie in these planes with a potassium atom symmetrically above and below the centre of each.

From the co-ordinates listed in table VI, it was possible to calculate the interatomic distances. The values obtained for the $0-0$ distances in the silica tetrahedra (fig. 5) are: $A B=2 \cdot 49, A C=A D=2 \cdot 64$, $B C=B D=2.53, C D=2.77 \AA$. The average of these is $2.60 \AA$., which is the generally accepted $0-0$ bond distance in these tetrahedra. Although the maximum error in these distances is $0 \cdot 1 \AA$., the value of $C D$ is still significantly large.

The $\mathrm{Si}-\mathrm{O}$ distances obtained are: $S A=1 \cdot 60, \quad S B=1 \cdot 58, \quad S C=$ $S D=1 \cdot 60 \AA$. The accepted value ${ }^{1}$ is $1 \cdot 60 \AA$. and the close agreement of the measured bond lengths tends to support the large $0-0$ distance quoted for $C D$. The $\mathrm{Zr}-\mathrm{O}$ distance is $2.07 \AA$. Goldschmidt radii for zirconium and oxygen would suggest this value to be about $2 \cdot 17 \AA$. The $\mathrm{K}-\mathrm{K}$ distance was found to be $3 \cdot 30 \AA$. as compared with Goldschmidt's distance of about $2.7 \AA$. Each potassium atom is surrounded by nine oxygen atoms; three at a distance of 2.82 , three at 3.06 , and three at $3 \cdot 44 \AA$. Goldschmidt radii suggest that these figures could be as low as $2.65 \AA$.

## Conclusion.

The complete structure of wadeite has been determined by X-ray methods. It has the space group, $P 6_{3} / m$, unit-cell dimensions

$$
a .6 \cdot 893 \pm 0.004, \quad c 10.172 \pm 0.002 \AA
$$

and may be represented ideally by the formula $\mathrm{Zr}_{2} \mathrm{~K}_{4} \mathrm{Si}_{6} \mathrm{O}_{18}$.
The fundamental structural unit is the $\left(\mathrm{Si}_{3} \mathrm{O}_{9}\right)^{6-}$ silicon-oxygen ring. The only previously known silicate mineral to be built up on this unit is benitoite ${ }^{2}$ from San Benito County, California, having the formula $\mathrm{BaTiSi}_{3} \mathrm{O}_{9}$ and space group $P \overline{6} c 2\left(D_{3 h}^{2}\right)$.

There is evidence of some distortion in the individual silica tetrahedra. This distortion causes two-thirds of the oxygen atoms in wadeite to be closer to the positively charged zirconium and potassium ions than

[^1]would be the case for regular tetrahedra, and yet it does not alter the $\mathrm{Si}-\mathrm{O}$ distances or the average $\mathrm{O}-\mathrm{O}$ distances in each tetrahedron.

Acknowledgements.-The author is indebted to Professor R. T. Prider who supplied the crystal of wadeite, to Mr. G. B. Robertson whose Geiger-counter equipment was used, to Mr. J. Shearer for helpful advice, and in particular to Professor C. J. B. Clews for his patient and helpful supervision. The work was made possible by a grant from the University Research Grants Committee.


[^0]:    ${ }^{1}$ O. Weisz, W. Cochran, and W. F. Cole, Acta Cryst., 1948, vol. 1, p. 83. [M.A.

[^1]:    ${ }^{1}$ J. V. Smith, Acta Cryst., 1954, vol. 7, p. 479. [M.A. 12-430.]
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