New techniques for the universal stage. *II. The determination of 2V when only one optic axis is accessible.*

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Summary. When one optic axis of a biaxial crystal has been located accurately by the usual orthoscopic or conoscopic methods, the second may be located by determining the extinction positions at normal incidence and applying the Biot–Fresnel construction. When use is made of the extinction directions associated with two or more wave normals inclined to the section, this method gives more reliable values of the optic axial angle than can be obtained by the doubling of the angle V over the pole of a bisectrix. If the optic axes have been located with sufficient accuracy by this procedure, then this also determines the orientation of the indicatrix.

When one optic axis and a bisectrix of a biaxial crystal have been located, it is customary to obtain the second optic axis by doubling the angle V over the bisectrix. This procedure, to which resort must frequently be made when the optic axial angle is large, is liable to give inaccurate values for $2V$ (errors of 2° to 5° are not uncommon), because the position of the first optic axis, or the bisectrix, or possibly both, may not have been determined with the requisite accuracy.

As the Biot–Fresnel construction may be used to determine the two vibration directions associated with any given wave normal within a biaxial crystal when the orientation of the wave normal is known relative to the optic axes, it may be used to check on the accuracy of the observations when both optic axes have been determined. This method can be applied if the two extinction directions associated with a given wave normal are known. For this purpose it is customary to use the wave travelling normally to the section. The extinction directions then would lie in the plane of the section (Hallimond, 1950 and 1953). Fig. 1 shows how this procedure is carried out on a stereographic projection.

This application of the Biot–Fresnel construction becomes inaccurate when the optic axial plane is nearly vertical and indeterminate when this plane is vertical. When the plane is nearly vertical, a very slight error in the determination of the first optic axis, the optic axial plane, or the extinction positions, can lead to a very considerable error in the derived position of the second optic axis. By inclining the section, however, this difficulty can be overcome (Hallimond, 1953), as several more suitable wave normals inclined to the plane of the section may be chosen and their associated extinction directions determined. In this way greater accuracy may be obtained and the method will always provide a satisfactory check on the location of the second optic axis when one bisectrix and one or both optic axes have been determined in the usual way.

The accurate location of the first optic axis. If one optic axis of a biaxial crystal can be determined accurately, the Biot–Fresnel construction may be used to determine the second optic axis, and hence the value of $2V$, with no other additional information than the pair of mutually perpendicular extinction directions associated with each of three or more wave normals. In order to save time in the choice of these wave normals, as they have to be sufficiently distant from the optic axial plane, it is convenient, though not necessary, to know the approximate orientation of this plane.

The success of this procedure depends greatly on the accuracy with which the first optic axis has been located. It is recommended, therefore, that after the optic axis has been made as vertical as possible by the orthoscopic procedure, an attempt should be made to locate its position more accurately by means of an interference figure. This can often be used with minerals of moderate birefringence, even if only an imperfect figure is obtained in other positions, for the isogyres are narrowest and most clearly defined in the vicinity of an optic axis. Consequently, there is a much greater chance of obtaining a useful interference figure from a grain in a thin section when in this orientation. The point of emergence of the optic axis can be recognized as the point about which the isogyre rotates when the microscope stage is revolved. The optic axis should be brought into coincidence with the axis of the microscope. If the optical system has been adjusted correctly, this latter direction should coincide with the centre of the conoscopic ring mark engraved on the back focal plane of the objective. If no such ring mark is provided, the intersection of the cross wires may be used for this purpose if the Bertrand lens has been centred correctly.

If the first optic axis cannot be located by convergent light methods,

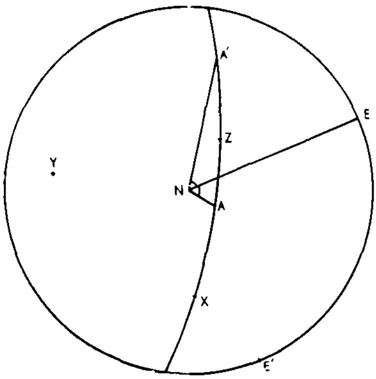

FIG. 1.

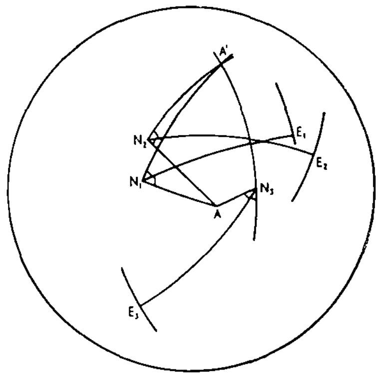

FIG. 2.

FIG. 1. The use of the Biot-Fresnel construction for locating the second optic axis. X, Y, and Z are the principal vibration directions, A is the optic axis which has been determined directly, E and E' are the extinction directions associated with the plane wave travelling normally to the section. An angle equal to $\widehat{ANE}$ is constructed on the opposite side of NE. The intersection of the great circle so obtained with the circle XAZ is the pole A' of the second optic axis. As a check, the arcs AZ and ZA' should be equal.

FIG. 2. The Biot–Fresnel construction applied to wave normals inclined to the section. (Crystal of labradorite from porphyritic basalt, St. John's Point, Co. Down.) One optic axis only, A, has been determined previously. By means of the three wave normals N_1, N_2, N_3, with their corresponding extinction directions E_1, E_2, E_3, the second optic axis is determined. As in fig. 1 an angle equal to the angle $\widehat{AN_1E_1}$ is constructed on the opposite side of N_1E_1. The same procedure is carried out using N_2 and N_3 with their corresponding extinction directions. The three great circles thus obtained intersect at A', the second optic axis.

use can be made of the 'optical curves' introduced by Fedorov (1896) and summarized by Johannsen (1918).

These optical curves may be obtained by the following procedure: All axes of the universal stage are set at zero, the nicols are crossed, and the A_4-axis is set parallel to the vibration plane of the polarizer or analyser. At successive inclinations of A_4, 0°, 10°, 20°, &c., N. and S., the crystal is set to extinction by a rotation of A_1. The wave normals corresponding to the pairs of readings (α_1, α_4) are then plotted on a stereogram. The locus of these points is the optical curve. These optical curves, in which wave-normals are plotted, must not be confused with the extinction curves described by Joel and Muir (1958), in which vibration directions are plotted.

Further optical curves are now obtained by repeating the same procedure for different positions of A_4 relative to the crossed nicols. All such optical curves should intersect at the poles of the optic axes (and also at the pole of the normal to the section). If the accessible optic axis has first been located approximately, it is necessary to determine only those portions of the optical curves near this optic axis.

An alternative orthoscopic method of refining the position of the accessible optic axis is given by Turner (1942).

The determination of the second optic axis. Fig. 2 shows the extinction directions associated with three inclined wave normals plotted on a stereographic projection. Details of the plotting procedure for extinction directions obtained from wave normals inclined to the section are given by Joel and Muir (1958), pp. 871–874. Fig. 2 shows also how the different great circles (one great circle for each wave normal), drawn to locate the second optic axis, should intersect to give its position. In theory, the intersection of two such great circles should be sufficient to fix the position of this optic axis, but a good check can be obtained by using three or more of them. If these great circles depart significantly from a unique intersection, it must follow either that one or more of the pairs of extinction directions have been determined incorrectly, or that the first optic axis has not been located with sufficient accuracy.

Corrections. This method is most easily applied when no correction is required for refractive index difference between crystal and hemispheres. This applies for instance when both the maximum birefringence $\gamma-\alpha$ and the difference $|n_c-n_h|$ between the average index of the crystal n_c and that of the hemispheres n_h are less than about 0·03.

Where the necessity for refractive index correction arises, a distinction must be made between minerals of low and high birefringence:

With minerals of low birefringence, $\gamma-\alpha \ll |n_c-n_h|$, and the average index n_c of the crystal is used to estimate the correction. This correction is then applied to the pole of the wave normal in the usual way, radially towards or away from the centre of the stereogram, before actually plotting the extinction directions. These are then inserted along the corrected great circle.

With minerals of high birefringence, significant errors may be incurred by using an assumed average index for estimating the correction. Ideally, in this case, the correction to the wave normal should be applied separately for the two vibrations, each with its correct refractive index. If these cannot be estimated by means of an approximate knowledge of the principal refractive indices and the orientation of the indicatrix, or by any other means, the average index n_c is still used. The errors involved in this approximation may be reduced by determining the extinction directions for a larger and randomly selected set of wave normals.

Conclusions. It is worth while stressing that the reliability of this method of determining the value of $2V$ lies in the fact that several arcs are required to intersect on (or very near to) a single point, which provides a good check. The same can be said of Fedorov's optical-curves method for determining the optic axis that is directly accessible. The combination of Fedorov's method and the one described in this paper should therefore lead to a determination of the two optic axes of great accuracy, limited only by the number of graphical constructions one wishes to make—in other words, by the time available to the observer.

It is interesting also to note that not only does this method lead to the determination of the second optic axis, and hence of $2V$, but it may be used to determine the actual orientation of the indicatrix quite independently of any other of the usual methods. The usefulness of this last application will depend, of course, on whether or not the error in the determination of the optic axes will be smaller than that of the conventional determination of the symmetry planes of the indicatrix, which may happen in some cases. Alternatively, it may be used as a check on the determination of the principal axes of the indicatrix.

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