A new method for the absolute measurement of reflectivity

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Summary. The theory of a new method for the determination of reflectivity at truly normal incidence is described. A parallel light beam falls on the specimen after passing through a glass cube with semi-silvered diagonal mounted at the centre of an optical goniometer. The various reflected beams are measured by a photomultiplier fixed to the telescope of the goniometer. Experiment has proved the validity of the method and the spectral reflectivity of pyrite in air and in oil has been investigated.

MEASUREMENTS of the reflectivity of opaque minerals for identification purposes are carried out by comparison with standards on which absolute measurements have been made. The available absolute methods are limited in number and suffer from two disadvantages. Firstly, the reflectivity at truly normal incidence is not determined but rather the reflectivity at angles of incidence up to 10 degrees. Secondly, it is not usually possible to determine the reflectivity of specimens in an immersion medium other than air. These disadvantages can apparently be overcome by a method based on the following fairly simple principles.

In fig. 1, PP is a glass slip of reflectivity R (for oblique incidence) placed in a vertical plane at the centre of a horizontal-circle optical goniometer, at 45° to a plane-polarized parallel light beam of intensity I_0 whose vibration direction is parallel to the surface of the glass slip so that no rotation of the plane of polarization will occur on reflection. The surface QQ of the specimen whose reflectivity S is to be determined is set up at right angles to the light beam. If a sensitive device for measuring light intensity is now attached to the goniometer telescope, the intensities $I_0SR(1-R)$ and I_0R can be measured. The specimen and glass slip are then removed and the intensity I_0 of the direct beam can be measured. R and S can then be calculated. The device for measuring intensity must be

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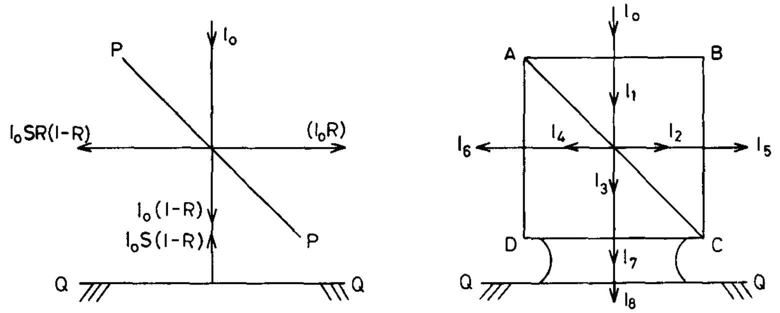
very sensitive as the final reflected intensity could differ by a factor of around 300 from the original intensity, and its response must be linear over this range. However, photomultipliers at present available should be able to fulfil these requirements.



FIGS. 1 and 2. FIG. 1 (left): Intensity relations with an ideal glass-slip vertical illuminator. FIG. 2 (right): Intensity relations with a vertical illuminator consisting of a glass cube with partially reflecting diagonal and oil immersion.

The simple relationships described above cannot be realized in practice because of internal reflections within the glass slip and displacement of the light beam, but the use of a glass cube with a very thin reflecting diagonal such as is used in certain types of optical apparatus overcomes some of the disadvantages. There will still be internal reflections between the faces of the cube, but because of normal incidence these can be allowed for in calculation. The diagonal of the cube has a much higher reflectivity than the simple glass slip, so reducing the problem of measurement. Fig. 2 shows the general case of a specimen whose reflectivity at normal incidence when immersed in oil is to be determined.

I_x denotes the intensity of light travelling in the direction shown by the arrow. J_x denotes the intensity of light travelling along the same path but in the opposite direction to I_x. Both I_x and J_x may involve several components due to multiple reflections. The space between the face CD of the cube and the specimen surface QQ is filled with a liquid of refractive index n. The thickness of the layer is greatly exaggerated in the diagram for clarity. Let the various reflection coefficients for normal incidence be: r for the glass-air interface, w for the glass-oil interface, S for the specimen in air, σ for the specimen in oil, whilst for oblique incidence on the diagonal AC let ρ be the reflection coefficient and θ the transmission coefficient (absorption in this surface cannot be neglected; $\rho+\theta \neq 1$).

Of these quantities, w can be calculated from r knowing n, leaving five to be determined. For this purpose, the following ratios can be determined experimentally: I_5/I_0, I_6/I_0, I'_5/I_0, I'_6/I_0, I'_7/I_0, where the prime indicates a reading with the sample QQ removed and J_7 consequently zero.

The following equations can now be derived for light leaving:

$$\begin{array}{lll}
\text{cube face } AB: & I_1 = rJ_1+(1-r)I_0, & J_0 = rI_0+(1-r)J_1 \\
BC: & I_5 = rJ_5+(1-r)J_2, & J_2 = rI_2+(1-r)J_5 \\
CD: & I_7 = wJ_7+(1-w)I_3, & J_3 = wI_3+(1-w)J_7 \\
DA: & I_6 = rJ_6+(1-r)I_4, & J_4 = rI_4+(1-r)J_6 \\
\text{diagonal } AC: & I_2 = \rho I_1+\theta J_4, & J_1 = \rho J_2+\theta J_3 \\
 & I_3 = \rho J_4+\theta I_1 & \\
 & I_4 = \rho J_3+\theta J_2 &
\end{array}$$

Assuming no internal reflection from the specimen and no appreciable back reflection from the measuring instrument, then $J_5 = J_6 = J_8 = 0$, so that $J_2 = rI_2$, $J_4 = rI_4$, $I_5 = (1-r)\,I_2$, $I_6 = (1-r)\,I_4$, and $J_7 = \sigma I_7$ (in oil) or SI_7 (in air).

By elimination the following intensity ratios are obtained as functions of r, ρ, θ, and σ:

$$I_6/I_0 = \rho\theta(1-r)^2\{(1-w\sigma)(r+w)+\sigma(1-w)^2\}/M$$

$$I_7/I_0 = \theta(1-r)(1-w)(1+r^2\rho^2-r^2\theta^2)/M$$

$$I_5/I_6 = \{(1-w\sigma)(1-rw\rho^2+rw\theta^2)+r\sigma(1-w)^2(\theta^2-\rho^2)\}/N,$$

where

$$\begin{aligned} M = {} & (1-w\sigma)\{(1-rw\rho^2-r^2\theta^2)^2-r^2\rho^2\theta^2(r+w)^2\}- \\ & -r\sigma(1-w)^2\{(1-rw\rho^2-r^2\theta^2)(\theta^2+\rho^2)+2r^2\rho^2\theta^2(r+w)\} \end{aligned}$$

$$N = \theta\{(1-w\sigma)(r+w)+\sigma(1-w)^2\}.$$

The second of these equations (I_7/I_0) is of use only when the specimen QQ is removed, when $\sigma = 0$, $w = r$ and the corresponding value I'_7 can then be measured.

The equations then reduce to:

$$I'_6/I_0 = 2r\rho\theta(1-r)^2/\{(1-r^2\rho^2-r^2\theta^2)^2-4r^4\rho^2\theta^2\}$$

$$I'_7/I_0 = \theta(1-r)^2(1+r^2\rho^2-r^2\theta^2)/\{(1-r^2\rho^2-r^2\theta^2)^2-4r^4\rho^2\theta^2\}$$

$$I'_5/I'_6 = \{1-r^2\rho^2+r^2\theta^2\}/2r\theta.$$

By elimination amongst these equations the following expressions for ρ, θ, and r in terms of the observed intensity ratios are obtained:

$$\rho = 1/W+P/2\pm\sqrt{(P/W+P^2/4)}, \quad \theta = \rho\sqrt{Z}, \quad r = 1/W\rho,$$

where

$$X = I'_5/I'_6, \quad Y = I'_7/I'_6, \quad Z = (Y^2-1)/(X^2-1), \quad W = X\sqrt{Z}+Y,$$

and $P = (I'_7/I_0)\{(1-1/W^2-Z/W^2)^2-4Z/W^4\}/(1+1/W^2-Z/W^2)\sqrt{Z}.$

The required value, σ, of the reflectivity of the specimen in oil is now given by

$$\sigma = (B-DI_5/I_6)/(I_5/I_6-A),$$

where

$$A = \{r(1-w)^2(\theta^2-\rho^2)-w(1-rw\rho^2+rw\theta^2)\}/\theta\{(1-w)^2-w(r+w)\},$$

$$B = \{1-rw\rho^2+rw\theta^2\}/\theta\{(1-w)^2-w(r+w)\},$$

$$D = (r+w)/\{(1-w)^2-w(r+w)\}.$$

In these expressions, the value of w is calculated knowing r and n, the refractive index of the immersion oil. For measurements in air the above expressions can be simplified by writing $w = r$ and $\sigma = S$.

For the glass used in an experimental test of the method, r is about 0·04 and ρ and θ are about 0·5. It would seem therefore that terms in the foregoing equations involving r^4 will be negligible, and since $r^2\rho^2$ is almost equal to $r^2\theta^2$, the term $1+r^2\rho^2-r^2\theta^2$ will not differ significantly from unity. With these assumptions the following simplified expressions may be used for calculation:

$$r = 1+Q/2\pm\sqrt{(Q+Q^2/4)}, \quad \theta = I'_6/2rI'_5, \quad \rho = I'_6/2rI'_7,$$

where $$Q = 2I'_7I'_5[1-\tfrac{1}{4}\{(I'_6/I'_7)^2-(I'_6/I'_5)^2\}]^2/I_0I'_6;$$

and the reflectivity S of the specimen in air is then given by

$$S = (B'-D'I_5/I_6)/(I_5/I_6-A'),$$

where $$A' = r\{(1-r)^2(\theta^2-\rho^2)-1\}/\theta\{(1-r)^2-2r^2\},$$

$$B' = 1/\theta\{(1-r)^2-2r^2\},$$

$$D' = 2r/\{(1-r)^2-2r^2\}.$$

Although the calculations required to derive results from the experimental observations are rather involved, they can be carried out in about 30 minutes using a desk calculator. In practice it is preferable to write a short computer programme and so avoid the risk of error. In view of the complexity of the calculations required and the occurrence of high powers of the various quantities, it is obvious that large errors will occur if the experimental results are not of sufficient accuracy. The experimental feasibility of the suggested method was therefore tested.

The cube, of borosilicate glass, has a vacuum-deposited inconel layer forming the diagonal reflecting surface and is cemented with cellulose caprate. The maximum departure of interfacial angle from 90° is 7 minutes, measured on the optical goniometer used for the experiment.

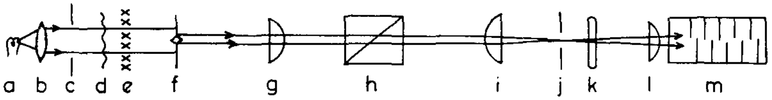


FIG. 3. Experimental arrangements: *a*, lamp filament; *b*, lamp condenser; *c*, lamp iris; *d*, interference filter; *e*, polaroid, vibration direction normal to page; *f*, collimator slit, ¼ mm diagonal; *g*, collimator lens; *h*, glass cube; *i*, telescope object lens; *j*, diaphragm; *k*, *l*, eyepiece lenses; *m*, photomultiplier

The cube was set up on the adjustable arcs of a Unicam horizontal-circle optical goniometer as shown in fig. 3. The photomultiplier and associated circuit described by Nichol (1962) were used for measurement of intensities, with the exception of a stabilized power pack in place of the accumulators to supply the light source and a digital voltmeter in place of the galvanometer to measure the output. The interference filter was of the continuous band type set to a wavelength of 589 mμ. The apparatus was switched on for 48 hours before making measurements.

Measurements were made in air on a polished specimen of Elba pyrite supplied by S. H. U. Bowie that had previously been measured by Nichol (1962) using Hallimond's apparatus (Hallimond, 1957) and on a polished specimen of silicon supplied by E. N. Cameron. For pyrite the mean reflectivity from five determinations was 55·8 with a standard error of 1·08. For silicon the mean reflectivity from twelve determinations was 38·4 with a standard error of 0·79. These compare with values of 54·8 for pyrite (Nichol, 1962) and 36·3 for silicon (E. N. Cameron, personal communication). The object of demonstrating the experimental feasibility of the method has therefore been achieved as the results fall within the expected experimental error for the existing apparatus. Some obvious minor improvements can be made, such as the use of a more intense light source, and it is hoped to obtain by this means results of greater precision. Preliminary results for the reflectivity of pyrite in oil showed a similar precision, but we feel that these should not be given before repeating the determination with the improved apparatus as there are no available figures against which they can be checked.

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