XXII. *Crystallographic Notes.* *By* L. Fletcher, *M.A., of the Mineral Department, British Museum; late Fellow of University College, Oxford*.*

[Plate VIII.]

X. *On Twins of Copper Pyrites.*

The memoir of Haidinger† on the Crystallisations of Copper Pyrites, published so long ago as the year 1822, was so exhaustive and withal so simple in its character that little seemed to be left to tempt the crystallographer to devote further study to this mineral; and in fact, with the exception of the papers of Sadebeck, whose early death all interested in the progress of mineralogy must so deeply deplore, and the confirmatory data in Groth's Catalogue of the Collection of the University of Strassburg, we have still no other information at our disposal. A study of these memoirs and of the various text-books of mineralogy leaves upon one's mind such a feeling of doubt as to the true statement of one of the laws of twin-growth, and that (as will be explained later) a law almost, if not quite, unique in the domain of crystallography, that, at the suggestion of Professor Maskelyne, the collection of copper pyrites in this Museum has been examined with a view to a possible settlement of the difficulty.

To get a clear idea of the present position, it is necessary to trace the history of this particular law from its first statement down to the present time.

In the above memoir of 1822, the twin-growths of copper pyrites were assigned by Haidinger to three distinct laws. In the first kind, the twin-plane, or plane of rotation, is a face of the octahedron $\{1\,1\,1\}$, and the composition-plane, or plane of junction, is generally parallel, but sometimes perpendicular, to the twin-plane: these growths correspond to the blende twins of the Cubic system. In the second, the twin-plane is a face of the octahedron $\{1\,0\,1\}$, and the composition-plane is *perpendicular* to the twin-plane. In the third, the twin-plane is a face of the prism $\{1\,1\,0\}$, and the individuals are interpenetrant. The second law is that with respect to which uncer-

* Read June 3, 1882.

† Memoirs of the Wernerian Society, vol. iv. p. 1, 1822.

tainty has arisen; and it is with this law that we have now to deal.

In the original memoir of 1822 the growth is so clearly described, and the law so distinctly expressed, that it is impossible to read the memoir and to mistake its meaning. Three years later, however, Haidinger published in the 'Edinburgh Journal of Science' a series of papers "On the Regular Composition of Crystallised Bodies," copiously illustrated with figures which have since found their way into almost every manual of the science; and in its natural place in the tetragonal (or pyramidal) system he describes once more, though briefly, the particular growth of copper pyrites which we are about to consider. There is little doubt that this secondary description has been the cause of serious misunderstanding.

On page 68 of vol. iii. (1825) we read as follows:—"Regular composition often also takes place in this species parallel to a plane of $P-1\{1\,0\,1\}$, or perpendicular to the terminal edges of $P\,\{1\,1\,1\}$. There are particularly two varieties of *this case* which in the present place deserve our attention. The individuals are either joined in pairs, or one central individual is surrounded by four others, added in the direction of all the edges of P. The product of the first, in the fundamental pyramid, would be fig. 30 [similar to fig. 6]. This has not yet been observed; but it will serve for explaining fig. 31, a variety of the form $P-\infty\,\{0\,0\,1\}$, $P-2\,\{1\,1\,2\}$, $P\,\{1\,1\,1\}$, $\frac{3}{2\sqrt{2}}P\,\{3\,0\,2\}$, and $\frac{3}{2\sqrt{2}}P+1\{3\,3\,2\}$; from the mines of the district of Siegen in Prussia. This and several other interesting varieties of forms from the same locality I have described on another occasion (*Mem. Wern. Soc.* vol. iv. part 1, p. 1, 1822), from specimens in the possession of Mr. Sack, of Bonn."

Taken by itself, this explanation might at first sight suggest the interpretation which seems to have been placed upon it by some crystallographers—namely, that there are twin-growths of copper pyrites in which the *plane of composition* is parallel to a plane of the octahedron $\{1\,0\,1\}$.

This interpretation can never have been intended by Haidinger. At the beginning of the series he had remarked that, for the precise definition of a twin-growth, two planes must be given:— first, the twin-plane, or plane of rotation, to indicate the relative *directions* of corresponding faces of the two individuals; and, secondly, the composition-plane, or plane of junction, to indicate their relative *positions*. Haidinger's initial sentence, "regular composition takes place parallel to a plane of $\{1\,0\,1\}$, or perpendicular to the terminal edges of $\{1\,1\,1\}$," may therefore refer either to the plane of twinning or to that of composition, and to this extent is indefinite. But seeing that no plane of $\{1\,0\,1\}$ is perpendicular to a terminal edge of $\{1\,1\,1\}$, the above sentence must indicate two distinct cases if the reference be to a composition-plane, while if the reference be to a twin-plane, only a single case is indicated; for, as will be shown later, a rotation through two right angles in a plane of the octahedron $\{1\,0\,1\}$ is crystallographically identical in its results with a rotation through two right angles "perpendicular" to (*i. e.* round) a terminal edge of the octahedron $\{1\,1\,1\}$. That Haidinger only recognizes a single case is evident from the italics in the next line of the above quotation. As a matter of fact, the figure of the twinned octahedron, repeated from his first paper, shows the plane of composition as perpendicular to the twin plane, though this is only evident after careful inspection. His final reference to the original memoir without calling attention to any deviation therefrom, shows that he was still in accord with the explanation there given.

From this we conclude that either a careful study of this later paper of Haidinger on the regular composition of crystallised bodies, or a simple reference to his original memoir on the crystallisations of copper pyrites, would have made clear the fact that Haidinger regarded the composition-plane as perpendicular to the twin-plane $(1\,0\,1)$, though we grant that at first glance his second paper might suggest that the composition-plane is, in some cases at least, parallel to the plane of twinning $(1\,0\,1)$.

In 1830, Naumann's well-known work* did in fact state the law differently from Haidinger, and assumed the plane of composition to be *parallel* to that of rotation. In his preface Naumann makes special mention of the help he had derived from Haidinger's series of papers on regular composition, and in his description of these particular growths adopts some of

* *Lehrbuch der reinen und angewandten Krystallographie*, 1830.

the figures there given. It seems quite impossible for this deviation from Haidinger to have been introduced wittingly; for attention is not directed to the difference in the explanations, and, further, it is unlikely that Naumann had been able, so soon after the publication of Haidinger's work, to undertake a special and minute study of this particular law.

We feel that the above reasoning shows clearly enough that Naumann's statement is a simple misinterpretation of that of Haidinger, and that this can only have resulted from the fact that Naumann referred to the secondary, and not to the original, explanation given by the author of the law.

As Naumann's 'Crystallography' became the recognized and universal text-book on the subject, this statement, though a mistaken one, has been extensively circulated, and appears probably in every text-book of the present day, though, as might have been expected, the explanation of 1822 is repeated in Haidinger's own manual of 1845.

In 1868* Sadebeck published the results of his study of specimens of copper pyrites (belonging chiefly to the Berlin collection), and in the explanation of the twins assumed the correctness of Naumann's statement of the law. In a second paper, published in the following year†, he gives an explanation of his position, so very brief and so clearly illustrative of the present difficulties that a translation is given here:—

"In my memoir on copper pyrites I have wrongly stated the law of twinning; for I have supposed the twin-plane to be also the composition-plane. According to this, one pair of tetrahedron-faces should meet in a salient angle of 1° 24', and the opposite pair in a reentrant angle of the same magnitude. After I had published the memoir, Haidinger informed me by letter that this was not the explanation he himself had given, as may be seen from his statement in the 'Edinburgh Journal of Science,' which runs thus:—'Composition takes place perpendicular to the terminal edges of P.' In consequence of this friendly private communication from so famed a Nestor of the science, I subjected the crystals again to a

* "Ueber die Krystallformen des Kupferkieses," *Zeit. d. deutsch. geolog. Gesellsch.* p. 605, vol. xx. 1868.

† "Allgemeines Gesetz für tetraëdrische Zwillingsbildung," *Zeit. d. deutsch. geolog. Gesellsch.* p. 642, vol. xxi. 1869.

careful study. The result was that I found it impossible to say whether the tetrahedron-faces actually coincided, or formed an angle of 1° 24′. This led me to retain my old view, since that law seemed to me a simpler one which regarded a plane of the form {1 0 1} as at once twin-plane and composition-plane. But if I now apply the general law for tetrahedral twins to this case, it follows that, as faces of the positive tetrahedron of the one individual are adjacent to faces of the positive tetrahedron of the other, the composition-plane is perpendicular to the twin-plane. The tetrahedron-faces therefore must really fall into a plane; and I hope that crystals may yet be found which will leave the matter beyond doubt."

From this we conclude:—first, that Sadebeck had followed Naumann, assuming his explanation to be that of Haidinger; secondly, that, owing to the practical difficulty of distinguishing between growths according to the two laws, he could come to no decision from simple examination of the specimens; and, thirdly, that he declared in favour of Haidinger's view merely that the law might not be at variance with a second law, which was true in certain cases, but of which the general application had not been proved.

Apparently before 1876 there was another change of view on the part of Sadebeck; for on page 82 of Rose and Sadebeck's 'Crystallography'*, where this law is briefly referred to, we read that "the individuals have a face of the form {1 0 1} for composition-plane;" and a footnote gives a reference to the first paper of Sadebeck, without stating whether or not he had since obtained that evidence of the incorrectness of Haidinger's explanation which was confessedly wanting so late as the time of publication of the second. The omission of any reference to the difficulty may have arisen from unwillingness to perplex the student of an elementary text-book.

The next mention of the law is made in Groth's Catalogue of the Strassburg Collection (1878). We there find that, "as for the regular growths of copper pyrites, the results of Sadebeck are quite confirmed by the specimens in the Strassburg collection;" and further on we read that these particular growths are symmetric twins about a plane of the form {1 0 1}. In

* Rose and Sadebeck's *Elemente der Krystallographie*, 1876.

other words, Sadebeck's *first* explanation, or that of Naumann, is accepted. There is no reference to the difficulty in which Sadebeck had found himself placed; indeed it is quite possible that the later explanation of Sadebeck, agreeing with that of Haidinger, had escaped notice owing to its having appeared in a paper dealing with a more general subject and having no reference to copper pyrites in its title: in any case no measurements are recorded which render it possible to distinguish between the two statements of the law.

In the latter part of the same year, according to a paper on haplohedral hemihedry *, either Sadebeck was in a state of doubt as to which is the correct explanation, or else he considered both correct; for we read as follows:—"If, on the one hand, a face of {1 0 1} be both twin-plane and composition-plane, the adjacent tetrahedron-faces form small salient or re-entrant angles; if, on the other hand, the composition-plane be perpendicular to the twin-plane, the tetrahedron-faces of the one are coincident with the adjacent faces of the other." The probability is that both are mentioned, not because Sadebeck believed them to be both true, but merely to show that in either case the position then being contended for was a tenable one; and in fact the position of the composition-plane has no further bearing on the argument of that paper.

It would at first sight appear that a difference of a right angle in the position of the plane of composition would manifest itself by angular differences in the twin sufficient to render any difficulty of distinction impossible; we shall therefore attempt to make quite clear what differences would be observed in growths characterized by such different laws.

Fig. 2 represents an equipoised octahedron {1 1 1} of copper pyrites, *a b c d* being one set of similar alternate faces, and $\alpha\ \beta\ \gamma\ \delta$ the other set respectively parallel to the first. This figure approaches very nearly to the regular octahedron of geometry and of the Cubic system, of which the faces are all equilateral triangles and the sections through the edges all squares: the difference therefrom was first made known by Haidinger in the memoir of 1822. Though $A\ B\ \bar{A}\ \bar{B}$, the basal section of an octahedron of copper pyrites, is a square, the sections $C\ A\ \bar{C}\ \bar{A}$,

* "Ueber geneigtflächige Hemiëdrie," *Zeit. d. deutsch. geolog. Gesellsch.* p. 601, vol. xxx. 1878.

$C B \bar{C} \bar{B}$ through the terminal edges are merely rhombs, the angles in the vertical axis being $90° 51'$, and in the horizontal axes $89° 9'$; the triangular faces are only isosceles, the vertical angle of each being $60° 29'$ and the basal angles $59° 45\frac{1}{2}'$.

In fig. 1, $a\, b\, c\, d$ and $\alpha\, \beta\, \gamma\, \delta$ are the stereographic projections of the points in which lines drawn through the centre parallel to the normals of the faces of the octahedron of fig. 2 would meet the sphere.

The faces of the form $\{1\,0\,1\}$ truncate the terminal edges of the octahedron $\{1\,1\,1\}$; $T\, \bar{T}\, Q\, \bar{Q}$, four of the poles of this form, are shown in fig. 1.

According to Haidinger's measurement, $2\,Ca$ is $108° 40'$, whence

$$\tan QC = \tan Ca \cos QCa = \tan Ca \cos 45°.$$

Thus

$$QC = 44° 34\tfrac{1}{2}',\ QT = 90° 51',\ \text{and}\ Qa = 35° 3\tfrac{3}{4}';$$

also

$$\cos Ta = \cos Qa \cos QT,$$

and

$$Ta = T\beta = 90° 41\tfrac{3}{4}',$$

while

$$T\alpha = Tb = 180° - 90° 41\tfrac{3}{4}' = 89° 18\tfrac{1}{4}'.$$

Fig. 3 represents a second crystal, with its faces $a_1\, b_1\, c_1\, d_1$ $\alpha_1\, \beta_1\, \gamma_1\, \delta_1$ parallel respectively to the faces $a\, b\, c\, d\, \alpha\, \beta\, \gamma\, \delta$ of fig. 2.

Both versions of the law assume that the plane of rotation or the twin-plane is a face of the form $\{1\,0\,1\}$. Let us take for the particular plane of rotation the plane $(1\,0\,\bar{1})$ which truncates the edge $\delta_1\, c_1$, or the edge $\delta\, c$, and is represented in the stereographic projection by its pole T.

On rotating the crystal represented in fig. 3 through two right angles round the normal $T\bar{T}$ to the plane $(1\,0\,\bar{1})$, its faces will take up positions represented in fig. 4; and the poles of the faces in this new arrangement are introduced in fig. 1 as $a_1\, b_1\, c_1\, d_1\, \alpha_1\, \beta_1\, \gamma_1\, \delta_1$. In the first place we may remark that as each of the pairs of faces $\delta_1\, c_1$ and $\gamma_1\, d_1$ is diagonally symmetrical to the line $T\bar{T}$ about which the rotation takes place, c_1 will after the rotation have a direction parallel to that belonging previously to the face δ_1, and still belonging to the face δ; and similarly the face d_1 will after the rotation have a direction parallel to that belonging previously to the face γ_1 and still belonging to γ. In other words, the faces $c_1 d_1 \gamma_1 \delta_1$ of the rotated octahedron shown in fig. 4 are respectively parallel to the faces $\delta \gamma d c$ of the octahedron of fig. 2. If the octahedron had been the regular one of geometry, not only these faces, but all the remaining faces and all the edges of the octahedron of fig. 4 would have been parallel to faces and edges of the octahedron shown in fig. 2. As the line $T \bar{T}$ bisecting $\bar{C} C$ is perpendicular to the edge $\bar{C} A$, and therefore not parallel to the edge $C A$, the point T will not be midway between A and $\bar{C}$; and thus, although the edge $A \bar{C}$ would be unchanged in direction by a rotation through two right angles about $T \bar{T}$, while B would be rotated to $\bar{B}$ and $\bar{B}$ to B, yet $\bar{C}$ would not be rotated exactly to A nor A exactly to $\bar{C}$; and the edge $C A$ of fig. 2 will thus not be parallel to $\bar{A}_1 \bar{C}_1$ of fig. 4, nor the edge $B A$ to the edge $\bar{B}_1 \bar{C}_1$. In fact, while the edge $A_1 \bar{C}_1$ is parallel to the edge $\bar{C} A$, the angle $A_1 \bar{C}_1 \bar{A}_1$ is, as stated above, $90° \, 51'$, and the angle $\bar{C} A C$ is $89° \, 9'$; whence the edges $\bar{C}_1 \bar{A}_1, C A$ must have a mutual inclination of $1° \, 42'$. Similarly, although the edge $A_1 \bar{C}_1$ is parallel to the edge $\bar{C} A$, and the plane $\bar{C} A B$ to the plane $A_1 \bar{C}_1 \bar{B}_1$, the angle $A_1 \bar{C}_1 \bar{B}_1$, as stated above, is $60° \, 29'$, and the angle $\bar{C} A B$ is $59° \, 45\frac{1}{2}'$, whence the edges $B A, \bar{B}_1 \bar{C}_1$ are mutually inclined at an angle of $43\frac{1}{2}'$.

The same results will follow, perhaps more simply, from a study of the stereographic projection of fig. 1; for as $T c = T \delta$, c_1 will be rotated into the position of δ, and δ_1 into the position of c, while γ_1 will be rotated to d, and d_1 to γ. With the other poles it will be different: thus α will rotate into the position α_1, where $T \alpha = T \alpha_1 = 89° \, 18\frac{1}{4}'$, and $a \alpha_1 = T a - T \alpha_1 = 90° \, 41\frac{3}{4}' - 89° \, 18\frac{1}{4}' = 1° \, 23\frac{1}{2}' = \beta b_1 = \alpha a_1 = b \beta_1$. Also $T \bar{Q}_1 = T \bar{Q} = 89° \, 9'$, whence $Q \bar{Q}_1 = 90° \, 51' - 89° \, 9' = 1° \, 42'$; and $T \bar{C} = T \bar{C}_1 = 44° \, 34\frac{1}{2}'$, whence $C \bar{C}_1 = 90° \, 51'$.

Next, let M be a point bisecting the arc $Q \bar{Q}_1$; the poles of the two octahedra will be symmetrically disposed to the twin-plane, represented in the projection by the line $\bar{M} B M$; for $T M = T B = 90°$, and the arcs $T b_1 \; T \alpha_1 \; T \alpha \; T b \; \bar{T} a \; \bar{T} \beta \; \bar{T} a_1 \; \bar{T} \beta_1$ are all equal. From this it will follow that $M a = M \beta = M \alpha_1 = M b_1$; also that $M a_1 = M \beta_1 = M b = M \alpha$, and that $M \gamma \; M \gamma_1 \; M d \; M d_1 \; M c \; M c_1 \; M \delta \; M \delta_1$, being all right angles, are equal to each other. The poles of the two individuals are thus not only symmetrical to the twin-plane $M B \overline{M}$, but also to the plane $T B \overline{T}$ at right angles with it, represented by an irrational symbol approximating to (100 0 99), and thus not a crystalloid plane. The same arrangement of poles might therefore be obtained by rotating the original octahedron about the line $M \overline{M}$ parallel to the tangent of the circle at T, and therefore to a "terminal edge" of the octahedron {1 1 1}. It may be remarked that, although the same arrangement of poles will be obtained by rotation about the lines $T \overline{T}$, $M \overline{M}$, the lettering will not be identical in the two cases; but as in both cases the planes which are quite or nearly coincident in direction are always represented in the one individual by italic letters and in the other by greek, there will be no crystallographic difference in the results of these two methods of derivation. This proves that, as has been mentioned above, Haidinger's statement, "regular composition often takes place parallel to a plane of {1 0 1} or perpendicular to the terminal edges of {1 1 1}," indicates only a single case if the reference be to the plane of rotation.

Haidinger's law might therefore be equally well expressed in the two following ways:—

I. Twin-plane a face of the form {1 0 1}; composition-plane perpendicular to the twin-plane.

II. Twin-axis a terminal edge of {1 1 1}; composition-plane parallel to the twin-plane.

We are now in a position to discuss the difference of growth which will be produced by a variation in the position of the plane of composition. As the line $T \overline{T}$ is perpendicular to the edges $A \overline{C}$ $C \overline{A}$ and passes through the centre of the crystal, the rhombs $T B \overline{T} \overline{B}$, $T B_1 \overline{T} \overline{B}_1$ of figs. 2 and 3 will be perpendicular to the twin-plane, and will each represent the composition-plane of Haidinger. If, now, by simple translation without rotation the half $T B \overline{T} \overline{B} A C$ above the rhomb of fig. 2 be associated with the half $T \overline{B}_1 \overline{T} B_1 A_1 C_1$ below the rhomb of fig. 4 in such a way that the two rhombs coincide, we get the composition represented in fig. 5. This figure will therefore be that of a growth according to the law of Haidinger. For convenience of direct comparison with the results following from the law as given by Naumann and Sadebeck, the same growth as fig. 5 is shown in fig. 6 as it would be seen either after a rotation of the whole figure through two right angles about the vertical axis, or by an eye placed at the back of the paper.

Again, let the rhombs $\mathrm{M\,B\,\overline{M}\,\overline{B}}$, $\mathrm{M\,B_1\overline{M}\,\overline{B}_1}$ of figs. 2 and 3 be the traces on the octahedron-faces of a plane parallel to the twin-plane, and therefore the composition-plane of Naumann. If now, just as before, the half $\mathrm{M\,B\,\overline{M}\,\overline{B}\,C\,\overline{A}}$ above the rhomb of fig. 2 be associated with the half $\mathrm{\overline{M}\,\overline{B}_1\,M\,B_1\,A_1\,\overline{C}_1}$ below the rhomb of fig. 4 in such a way that the rhombs coincide, we get the composition represented in fig. 7. This figure will therefore be that of a growth according to the law of Naumann.

With the help of the stereographic projection of fig. 1 we can now investigate the differences of the growths represented respectively in figs. 6 and 7.

Fig. 6. *Haidinger's law.*	Fig. 7. *Naumann's law.*
$\left.\begin{array}{l} d\gamma_1=\gamma d_1=0, \\ c\delta_1=\delta c_1=0. \end{array}\right\}$	$\left\{\begin{array}{l} a\,\alpha_1=\beta\,b_1=\text{salient } 1^\circ\ 23\tfrac{1}{2}', \\ b\,\beta_1=\alpha\,a_1=\text{reentrant } 1^\circ\ 23\tfrac{1}{2}'. \end{array}\right.$
Edges $\mathrm{C\overline{T}}$, $\mathrm{\overline{T}C_1}$ coincident, " $\mathrm{A\,T}$, $\mathrm{T\,A_1}$ "	Angle $\mathrm{C\,M\,.\,M\,\overline{C}_1}$ = salient $1^\circ\ 42'$, " $\mathrm{\overline{A}\,\overline{M}\,.\,\overline{M}\,A_1}$ = reentrant $1^\circ\ 42$
Angle between the truncating planes of C and $\mathrm{C_1}$ $=89^\circ\ 9'$.	Angle between the truncating planes of C and $\mathrm{\overline{C}_1}$ $=90^\circ\ 51'$.
The faces $a\,\beta\,a_1\,\beta_1$ are not in a zone.	The faces $d\,\gamma\,c_1\,\delta_1$ are in a zone.

We have seen that in the projection the plane of composition is a plane of symmetry to the poles of the two individuals; and we further perceive that in each case the plane of composition is a plane of symmetry to the faces actually shown by the twin-growths; and the angles of the upper half of fig. 6 are exactly equal to the similarly disposed angles of the upper half of fig. 7, and the angles of the lower half of fig. 6 to the similarly disposed angles of the lower half of fig. 7, the only difference in the growths being the relation of the upper to the lower half.

Of the specimens of copper pyrites in this collection, one figured by Haidinger himself in the 'Catalogue of the Allan-Greg Collection' (now in the possession of the British

Museum), though probably not one of the original specimens of the memoir of 1822, offered itself as the most likely to afford a satisfactory solution of the difficulty. In this specimen, which comes from Freiberg, the twin-growths are disposed parallel to each other on galena, and are associated with quartz, chalybite, and calcite. The length of a side of the triangular faces is about 1·5 millim. Haidinger's drawing to illustrate this specimen is virtually the same as fig. 9 (copied from his paper of 1825), the only difference being that the faces of $e\{1\,0\,1\}$ being almost linear are not shown in the Catalogue. This figure represents the rotation as taking place about each of the normals to the four upper faces of $\{1\,0\,1\}$ of the central individual, of which both the upper and the lower halves are present. We may remark that a complex growth of this perfect kind would be explained by the law of Naumann equally with that of Haidinger, seeing that in the lower half of the regular composition the various planes of junction are represented as parallel to the corresponding twin-planes, and in the upper half as perpendicular to them. As none of the growths have an all-round development, the figure represents the growth in theoretical perfection rather than as actually existent; in fact the actual habit is more nearly shown in fig. 10, which at the same time will serve to give an idea of the striation to be observed on the faces.

A crystal from this specimen appeared to show that the triad of octahedron-faces $o\,o_1\,o_2$ did not quite coincide, as according to Haidinger's explanation should be the case; but still no satisfactory measurement of the angle could be obtained. The reentrant angle lying in the zone $c\,c_1$ at the junction of two individuals could, however, be determined with very fair precision, although the faces are finely striated parallel to their edge of intersection with each other. These planes were supposed by Haidinger to belong to the common form $z\{2\,0\,1\}$; and as the angle made by a plane of this octahedron with the adjacent face of the form $\{1\,0\,1\}$ is $18°\ 31'$, the reentrant angle according to his theory should be $37°\ 2'$. Actual measurement, however, gave $20°\ 9'$; and closer examination rendered it clear that on the crystal measured the faces belonged not to the form $z\{2\,0\,1\}$, but to another less common form $h\{3\,0\,2\}$, whilst smaller almost linear faces of $z\{2\,0\,1\}$ were to be seen lower down in the reentrant angle. The angle between (3 0 2) and (1 0 1) being 11° 20½′, the reentrant angle for this form should be, according to Haidinger's law, 22° 41′; on the other hand, according to Naumann's law, it should be 20° 59′ —that is, the difference of the inclinations of (3 0 2) and its parallel ($\bar{3}$ 0 $\bar{2}$) to the plane (1 0 $\bar{1}$). The measured angle was thus 50′ less than the angle calculated from Naumann's law; and the latter angle was itself 1° 42′ less than the one calculated according to that of Haidinger.

The difference between the calculated and measured angles is large, and in fact is so considerable that it can only be attributed either to the growth being not strictly a regular twin, or to a deviation from the fundamental angle as determined by Haidinger. As the extreme accuracy of Haidinger's measurement of the fundamental angle of copper pyrites had been confirmed both in the memoir of Sadebeck and in the Catalogue of the Strassburg collection, and also by Kokscharow*, and as, further, this particular crystal did not lend itself to a precise determination of any other angle than the reentrant one above mentioned, the result seemed very unsatisfactory, and for some time the examination was discontinued. Two years later it was resumed; and, happily, another crystal from the same specimen was found to give reflections so good that a precise measurement of the angle between two octahedron-planes belonging to the same individual and on opposite sides of c could be obtained. This was found to be 108° 17½′, a deviation of 22½′ from the angle as determined from the specimens previously measured. A less precise determination of the angle of a terminal edge gave as mean 69° 59¼′, the limiting values being 69° 54¾′ and 70° 0½′. From the same crystal the angle $o\,o_1$ was found by help of the δ eye-piece of a Fuess's goniometer (as improved by Websky) to be 2° 3′, instead of zero according to Haidinger, and 1° 23½′ according to Naumann and Sadebeck.

The whole difficulty had, however, now disappeared, as is shown by the following table of calculated and observed angles:—

* *Bull. Soc. St. Pét.* 1874, xix. p. 562.

If 001.101=	44° 34½′	44° 22½′	44° 22′	44° 21½′	Observed.
	Calculated.				
$\omega_2\,\omega'_2$	108° 40′	108° 17¼′	108° 16¼′	108° 15¼′	108° 17½′
$o\,\omega$	70 7½	69 56	69 55½	69 55	69° 54¾′–70° 0½′, mean 69° 59¼′
$o\,o_1$ Naum....	1 23½	2 3	2 4½	2 6	2° 3′
$o\,o_1$ Haid. ...	0 0	0 0	0 0	0 0	
$h\,h_1$ Naum....	20 59	20 12¾	20 10¾	20 8¾	20° 9′
$h\,h_1$ Haid. ...	22 41	22 42¾	22 42¾	22 42¾	

From a third crystal not quite so perfect, but still giving very good images, the angle $\omega_2\,\omega'_2$ was determined to be 108° 18¾′.

There can thus be no doubt that:—

1st. The growth is strictly regular.

2nd. The parametral angle differs from that determined from other specimens by previous observers, and is very nearly 44° 22′.

3rd. The twin-plane is a face of the form {1 0 1}. And

4th. The composition-plane is *parallel* (and not perpendicular) to the twin-plane.

At the suggestion of Prof. Maskelyne, a careful analysis of this specimen was made in the departmental laboratory, with the view of ascertaining to what extent this variation in the fundamental angle is attended by a difference from the chemical composition of ordinary copper pyrites. The following results were obtained by Dr. Walter Flight:—

	Observed. I.	Observed. II.	Calculated, $CuFeS_2$.
Copper . .	25·78	30·66	34·45
Iron . . .	35·16	34·11	30·57
Sulphur . .	37·52	[35·23 by diff.]	34·98
Arsenic . .	traces		
Quartz . .	0·28		
	98·74	100·00	100·00

No other metals were discovered, although carefully searched for. The specimen is thus found to contain a considerable and variable excess of FeS_2 over that of typical copper pyrites—the first analysis corresponding very nearly to

$Cu\,FeS_2 + \frac{1}{2}\,Fe\,S_2$, and the second to $Cu\,FeS_2 + \frac{1}{4}\,FeS_2$. As close examination of the specimen reveals the presence of included minute crystals of iron pyrites (and mispickel), it is possible that even in the second fragment, which was specially chosen for its approximate homogeneity, this excess of FeS_2 may be almost wholly due to mere admixture of iron pyrites.

Since the above determinations were made, the collection has been enriched by the acquisition of a superb Freiberg specimen of very much the same character as the one just described, but on a relatively colossal scale; for the length of side of the triangular face $o\,o_1\,o_2$ (and this is the smallest) is here not 1·5 but 24 millim. Owing to the absence of reentrant angles, the growth very much resembles a regular octahedron truncated by the faces of a cube, one of the octahedron-faces, however, showing in a very marked way the threefold composition of that part of the crystal.

A further examination of the collection has resulted in the finding of a specimen (from Pool mines near Redruth) which confirms the parallelism of the planes of composition and of twinning in the most satisfactory way. The crystals of copper pyrites have been here deposited on quartz crystals which they partly enclose: they thus have not an all-round development; in fact, there is a practical difficulty in determining more of the growth than is shown in fig. 11; but this, so far as it goes, falls little short of perfection (the dotted lines indicate the twin in theoretical completeness). The images from $\omega\,\omega_2$ were so well defined that the largest magnifier, α, of Fuess's instrument could be used, three different measurements at nearly normal incidence giving respectively for the value of this angle $1^\circ\ 23'$, $1^\circ\ 23\frac{1}{2}'$, and $1^\circ\ 23'$, while a measurement at almost grazing incidence, when the images were broader, gave $1^\circ\ 23\frac{1}{2}'$. The faces $o\,o_2$ were not so perfect; but still, with the δ eyepiece, good images were seen, from which two consecutive measurements of the angle gave $1^\circ\ 23'$ and $1^\circ\ 23\frac{1}{2}'$. Two measurements of $o\,o'$ gave respectively $108^\circ\ 39'\ 30''$ and $108^\circ\ 39'\ 45''$. The following table renders more evident the close correspondence of the observed and calculated angles.

Observed. Angles of one individual.		Observed. Symmetrically disposed angles of the other.		Calculated from $001.101 = 44° 34\frac{1}{2}'$.
$o\,\omega$	$70° \; 7\frac{1}{2}'$	$o_2\,\omega_2$	$70° \; 7\frac{1}{2}'$	$70° \; 7' \; 32''$
$\omega\,e$	$35° \; 3\frac{1}{2}'$	$\omega_2\,e_2$	$35° \; 4'$	$35° \; 3' \; 46''$
$\omega\,o'$	$70° \; 4\frac{3}{4}'$	$\omega_2\,o'_2$	$70° \; 12', 70° \; 4\frac{1}{2}'$	$70° \; 7' \; 32''$
		$o'_2\,e'_2$	$35° \; 4'$	$35° \; 3' \; 46''$
$o\,o'$	$108° \; 39\frac{5}{8}'$	$o_2\,o'_2$	$108° \; 37'$	$108° \; 40' \; 4''$
Also	$\omega\,\omega_2$	$1° \; 23', 1° \; 23\frac{1}{2}', 1° \; 23', 1° \; 23\frac{1}{2}'$		$1° \; 23' \; 30''$
	$o\,o_2$	$1° \; 23', 1° \; 23\frac{1}{2}'$		$1° \; 23' \; 30''$

The linear faces $e\{1\,0\,1\}$ and the minute faces of another octahedron $g\{2\,0\,3\}$, although present on the crystal, are not shown in the figure.

A chemical examination made by Dr. Flight shows that this specimen has a composition very nearly represented by the typical formula $CuFeS_2$. The following results were obtained:—

	Observed.	Calculated, $CuFeS_2$.
Copper	34·37	34·45
Iron	30·03	30·57
Sulphur	31·92	34·98
Quartz	4·19	
	100·51	100·00

Up to this point, for the sake of simplicity, a very important property of copper pyrites, its hemihedral structure, has been left as much as possible out of sight. It is found, however, that the faces of the octahedron $\{1\,1\,1\}$ of this mineral are not all similar, but must be regarded as belonging to two distinct tetrahedra: in the case of the first tetrahedron, for convenience of distinction termed the positive or o tetrahedron, the faces are rough or striated, and are sometimes coated with oxide of iron; on the other hand, the faces of the second or negative or ω tetrahedron are smooth, bright, free from this coating, and in general smaller than the former. From this it follows that the set of faces denoted above by the italic letters $a\;b\;c\;d$, though similar to each other, are distinct in physical character from those denoted by the greek letters $\alpha\;\beta\;\gamma\;\delta$; whence we infer that in such a growth as would be represented by fig. 7, where there has been a simple rotation of one individual through two right angles from a position of identical orientation with the other, and adjacent faces of the two individuals thus bear respectively italic and greek letters, the composition-plane will be a plane of geometrical, but not of physical symmetry. As, however, the correlative tetrahedra and also the correlative hemiscalenohedra are independent of each other, not only in surface-characteristics, but also in their presence on the crystal, even this geometrical symmetry could scarcely be expected in the actual twin-growth.

Now Sadebeck states that in the actual twin-growth the composition-plane is really a plane of symmetry not only to the geometrical, but to the physical peculiarities—the regular composition thus belonging to the class called by Groth "symmetric twins;" that instead of the faces of the octahedron which are parallel, or nearly so, in the two individuals belonging in one to the positive, and in the other to the negative, they really belong either both to the positive or both to the negative tetrahedron. To pass, therefore, to the actual twin from parallel orientation of the individuals, there must be, in addition to the rotation through two right angles round a normal to $(1\,0\,\bar{1})$, a *further rotation* of one of the two crystals either through two right angles about a normal to one of the faces of the prism $\{1\,1\,0\}$, or through a single right angle about the vertical axis parallel to the edges of this prism. Though this double rotation may be compounded into a single rotation round the normal to a face of the octahedron $\{1\,1\,1\}$, the angle of this single rotation will not be 180°, as is the case in other twins, but 119° 31′.

If this statement of Sadebeck be accepted as having a satisfactory foundation, the growth must be regarded as up to the present unique in character; for no other regular composition appears to have yet been discovered in which, starting from a parallel orientation, a double rotation is *absolutely* necessary for the representation of the relative disposition of the two individuals. There exist twin-growths of *tetartohedral* crystals, it is true, such as those of sodium chlorate and certain regular compositions of quartz, described by Prof. Groth, which are somewhat analogous in character; but they are capable of a more or less satisfactory representation by a simple rotation of one of the individuals through two right angles from a position where corresponding crystallographic lines of the right and left individuals are identical in direction; they are moreover intimately related to the directions of the crystallographic axes. As, however, it had been impossible for Sadebeck to convince himself, from simple examination of the specimens, that certain faces assumed by Haidinger to be parallel might not be inclined to each other at an angle of $1° 23\frac{1}{2}'$, it was possible to entertain a doubt as to the specimens being sufficiently well crystallised to allow of an absolute certainty in the distinction of the two tetrahedra; and as the law is so curious from its extreme rarity and simplicity, and so important in its bearing on the general question of twin-growth, about which there has lately been much discussion, it seemed desirable to place the law, if possible, beyond all suspicion.

The accuracy of Sadebeck's inference as to the disposition of the two tetrahedra in this twin-growth is confirmed in the most satisfactory manner by the specimens in this collection.

The Freiberg specimen of fig. 10 shows not only that the individuals are symmetrical to the plane of composition, but also that the differences of the two tetrahedra of each individual are too marked to allow of this symmetry of physical peculiarities being an accident of the growth.

The specimen from Pool mines (fig. 11) is even more satisfactory still; for the faces $\omega\,\omega_2$ which give such excellent images are perfectly smooth and bright, and remarkably different in aspect from the two dull and striated faces $o\,o_2$.

A further example is presented by a specimen (probably from the Trevannance mine, St. Agnes) shown in fig. 12, which the symmetry to the combination-plane and the extreme difference between the smooth and the deeply-striated tetrahedra render most convincing. The angle between these striations is so very definite that it can be measured with fair accuracy by means of a microscope; it was determined to be $120\frac{3}{4}°$, the angle calculated according to Naumann's law being $120° 28'$, and according to Haidinger's law $119° 31'$.

Finally, we may refer to fig. 8, representing a Cornwall specimen (now in the Museum) figured in 1825 by Haidinger himself in his memoir on the Regular Composition of Crystallised Bodies. Here the predominant form of each individual is a hemiscalenohedron; and this in each pair is symmetrically disposed to the plane of composition. Although this specimen is symmetrical in its habit, the planes *s* are so striated and rounded that it was found impossible to assign to them a definite symbol; they lie, however, in the zone defined by the symbol $[1\,1\,\bar{2}]$, and approximate to $\{3\,1\,2\}$.

We conclude, therefore, that there is no doubt of the actual existence of a kind of twin-growth which it is not possible to represent by a single rotation through two right angles from a position of parallel orientation of one of the individuals to the other—that for the representation of this growth an additional rotation is requisite, but that the simplest mode of representation is the one which regards the two individuals as symmetrical to a plane.

EXPLANATION OF PLATE VIII.

(To accentuate the differences in twin-growths according to the laws of Haidinger and Naumann, figs. 1–7 are drawn for a parametral angle $42\frac{1}{2}°$ instead of $44°\ 34\frac{1}{2}'$.)

Fig. 1. Stereographic projection of the poles of $\{1\,1\,1\}$, and of the same twinned about $T\,\bar{T}$, the normal to $(1\,0\,\bar{1})$.

Fig. 2. The octahedron $a\,b\,c\,d\,\alpha\,\beta\,\gamma\,\delta$ $\{1\,1\,1\}$.

Fig. 3. The octahedron $a_1\,b_1\,c_1\,d_1\,\alpha_1\,\beta_1\,\gamma_1\,\delta_1$ $\{1\,1\,1\}$, parallel to the last.

Fig. 4. The same turned through two right angles round $T\,\bar{T}$ the normal to $(1\,0\,\bar{1})$.

Fig. 5. Twin-growth of $\{1\,1\,1\}$, according to Haidinger's law.

Fig. 6. The same, viewed from the opposite side.

Fig. 7. Twin-growth of $\{1\,1\,1\}$, according to Naumann's law.

Fig. 8. Twin-growth, with faces *s* of a hemiscalenohedron or disphenoid (Haidinger, Edin. J. of Sc. 1825).

Fig. 9. Twin-growth of $\{1\,1\,1\}$ $\{0\,0\,1\}$ $\{1\,0\,1\}$ $\{2\,0\,1\}$ (Haidinger, Edin. J. of Sc. 1825).

Fig. 10. A similar twin-growth.

Fig. 11. A twin-crystal from Pool mines, near Redruth.

Fig. 12. A twin-crystal, probably from Trevannance mine, St. Agnes.

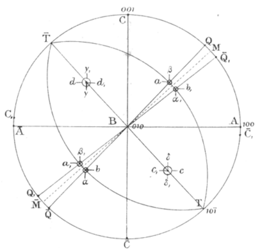

Fig: 1.

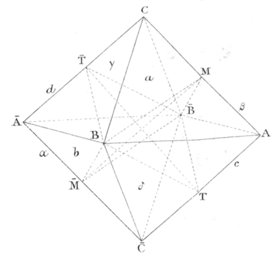

Fig: 2.

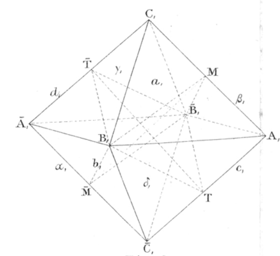

Fig: 3.

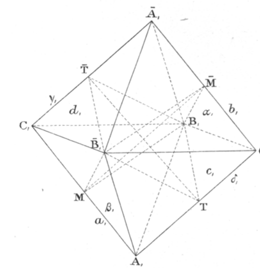

Fig: 4.

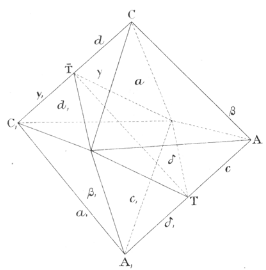

Fig: 5.

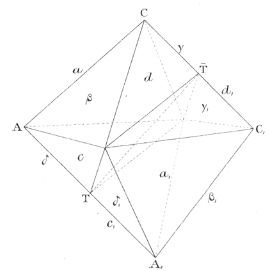

Fig: 6.

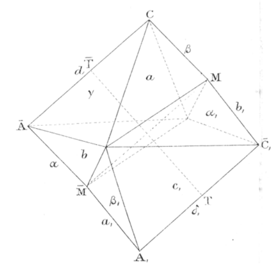

Fig: 7.

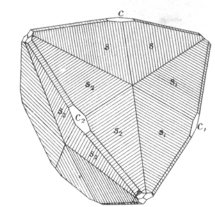

Fig: 8.

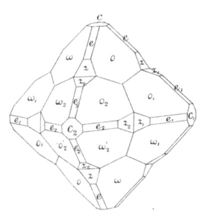

Fig: 9.

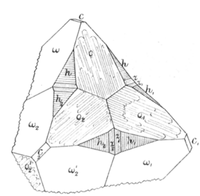

Fig: 10.

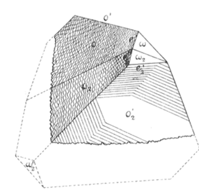

Fig: 11.

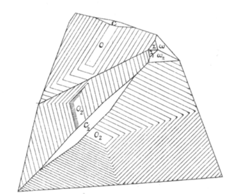

Fig: 12.

L. Fletcher del.

Mintern Bros. lith.

Copper Pyrites.