

and then to observe the reflected image alternately on both faces of the prism.

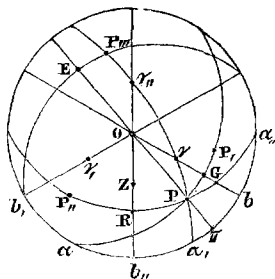
In order to be able to use the instrument for other purposes, both telescope and collimator can be shifted parallel to their axes and be fixed by the screws  $OO'$ . If the instrument is to be used as a spectroscope, one may put conveniently before the objectives of telescope and collimator prisms with direct vision.

### XV. Note on the Analysis of the Rhombohedral System.

By W. J. LEWIS, M.A., Fellow of Oriol College, Oxford\*.

THE methods followed by Professor Miller and most writers for obtaining the formulæ employed in determining the indices of a form in the rhombohedral system from the measured angles, or, conversely, the angles from the given indices, are, though elegant, difficult and perplexing. It occurred to me that they might be easily obtained by means of the anharmonic ratio of four poles in a zone applied to three known poles in one of the planes of symmetry, and a fourth pole whose position and indices can be directly connected with the poles of the form to be determined. This method brings out in a prominent manner the relation (2); a relation to be found in all the books, but so disguised and so little noticed as easily to be passed over, whereas from its simplicity, and from the fact that the angle involved in it is the first deduced from the measured angles of a scalenohedron, it contains a smaller error than any other equation.

The figure represents the stereographic projection of some of the principal poles and planes of a rhombohedral crystal, together with the poles  $P$  of a form  $\{hkl\}$  to be determined. The poles  $r$  are  $\{100\}$ ,  $o\{111\}$ ; therefore the poles  $b$  and  $a$  are  $\{2\bar{1}\bar{1}\}$  and  $\{01\bar{1}\}$  respectively. Let  $P$  be  $(hkl)$ ,  $P_{//}$ ,  $P_{..}$  the corresponding faces repeated over  $O\bar{b}$  and  $O b_{..}$ . Then  $P_{/}$  is  $(h\bar{l}k)$ , and



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$P_{//}$  ( $kh$ l). Let Q, R,  $\pi$  be the intersections of the pairs of zones  $[PP_{//}]$   $[Ob]$ ,  $[PP_{//}]$   $[Ob_{//}]$ ,  $[OP]$   $[bb_{//}]$  respectively. Then the indices of Q are  $(2h, k+l, k+l)$ , of R  $(h+k, h+k, 2l)$ , and of  $\pi$   $(2h-k-l, -h+2k-l, -h-k+2l)$ .

The anharmonic ratio of the poles  $a, b_{//}, \pi, b$  gives

$$\frac{\sin b\pi}{\sin \pi a} : \frac{\sin bb_{//}}{\sin b_{//}a} = \left[ \frac{b\pi}{\pi a} \right] : \left[ \frac{bb_{//}}{b_{//}a} \right] = \frac{k-l}{2h-k-l}$$

(Miller's 'Treatise on Crystallography,' p. 14). Hence

$$\tan b\pi = \tan \text{XOP} = \frac{(k-l)\sqrt{3}}{2h-k-l} \dots (1)$$

The anharmonic ratio of the poles O,  $r, Q, b$  gives, in a similar manner,

$$\frac{\tan OQ}{\tan Or} = \left[ \frac{OQ}{bQ} \right] : \left[ \frac{Or}{br} \right] = \frac{2h-k-l}{2(h+k+l)}$$

and writing D for the element Or, we have

$$\tan OQ = \frac{2h-k-l}{2(h+k+l)} \tan D \dots (2)$$

Similarly from the poles O, R,  $b_{//}$ , and  $z$  ( $2\ 2\ \bar{1}$ ) the dirhomboidal face of  $r_{//}$  we obtain

$$\frac{\tan OR}{\tan D} = \left[ \frac{RO}{Rb_{//}} \right] : \left[ \frac{zO}{zb_{//}} \right] = \frac{h+k-2l}{2(h+k+l)} \dots (2')$$

From the right-angled triangle POQ we have

$$\tan OP = \tan OQ \sec b\pi; \dots (A)$$

$\therefore$  from (1) and (2),

$$\tan^2 OP = \frac{(k-l)^2 + (l-h)^2 + (h-k)^2}{2(h+k+l)^2} \tan^2 D \dots (3)$$

The equation (2) is given by Professor Miller in his 'Treatise on Crystallography,' 1839, in the form

$$\tan PO \tan XO \cos \text{XOP} = \frac{2h-k-l}{h+k+l},$$

and in the equivalent form

$$2 \tan PO \cot OA \cos \text{XOP} = \frac{2h-k-l}{h+k+l},$$

the latter being the same as (2), with the sole difference that  $\tan OQ$  is replaced by its value given by equation (A). The form in which it is given by Professor Miller does not, how-

ever, bring out so prominently the simplicity and directness of the relation existing between the quantities involved in the equation and those given by observation.

As an illustration of the utility of equations (2) and (2'), let us take the determination of a scalenohedron on a mineral (such as calcite) whose elements are known. Measurement of two of the angles between adjacent faces suffices for the determination. If  $PP_1$  and  $PP_2$  are the two angles measured, we know the three sides of the triangle  $aPa_1$ ; and the angle  $Pab = bQ = \frac{\pi}{2} - OQ$  is the first quantity deduced from the measurements. Equation (2) then gives a simple equation in terms of the indices  $h, k, l$ . If  $PP_1$  or  $PP_2$  be given with the angle of the middle edge of the scalenohedron, we know the sides of the triangles  $aPa_1$  or  $a_2Pa_2$ . In the first case  $OQ$  is determined as before, in the second  $OR$ ; and we must employ (2) or (2') accordingly.

To complete the analysis, I need only point out that the relations connecting the indices of dirhomboidal forms can be most simply obtained by aid of the equations connecting the indices of a face with those of the zone in which it lies. Thus  $E$ , the face of the dirhomboidal form corresponding to  $P$ , lies in the zones  $[OP]$  and  $[b_1P_1]$ , whence its indices can be at once obtained, and all the geometrical relations connecting it with  $P$  can be proved. Professor Maskelyne has, I believe, already given this method of deducing the indices of the dirhomboidal form in his lectures at Oxford.

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#### XVI. *The Dilatation of Crystals on Change of Temperature.*

By L. FLETCHER, M.A., *Fellow of University College, Oxford,*  
*Assistant in the Mineralogical Department, British Museum.*

DURING the last fifty years a considerable amount of energy has been devoted by mathematicians and crystallographers to the investigation of the nature of the alteration of a crystal due to change of temperature, and it cannot even yet be said that the state of our knowledge is very satisfactory. It must be hoped that this short paper may serve to once more direct attention to this important and interesting question.