# The crystal structure of fresnoite, $\mathrm{Ba}_{2}(\mathrm{TiO}) \mathrm{Si}_{2} \mathrm{O}_{7}$ 

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## Auszug

Die Kristallstruktur von Fresnoit, $\mathrm{Ba}_{2}(\mathrm{TiO}) \mathrm{Si}_{2} \mathrm{O}_{7}$, wurde aus der PattersonProjektion $P(u v)$ bestimmt und nach der Methode der kleinsten Quadrate aus $h k l$-Interferenzen verfeinert. Eigentümlich für die Struktur ist die tetragonalpyramidale Umgebung des Titans. Schichten aus heterozyklischen fünfgliedrigen Ringen, die von tetragonalen Ti-O-Pyramiden und Si -O-Tetraedern gebildet sind, werden durch Ba-O-Bindungen zusammengehalten. Die Bindungen innerhalb der Schichten sind vorwiegend kovalent; zwischen den Schichten ist Ionenbindung vorherrsehend.

## Abstract

The structure of fresnoite, $\mathrm{Ba}_{2}(\mathrm{TiO}) \mathrm{Si}_{2} \mathrm{O}_{7}$, was determined from the Patterson projection $P(u v)$ and refined by the least-squares method using threedimensional single-crystal data. Its unique feature is the tetragonal-pyramidal coordination of titanium. Sheets made up of heterocyclic five-membered rings of Ti-O tetragonal pyramids and Si-O tetrahedra are held together by $\mathrm{Ba}-\mathrm{O}$ bonds. The bonds within the sheet are predominantly covalent, and the intersheet bonds are mainly ionic in character.

## 1. Introduction

Fresnoite, a mineral found in the sanbornite deposits of eastern Fresno County, California, was first described by Alfors et al. (1965). Based on a spectrographic analysis they report the composition

$$
\left(\mathrm{Ba}_{1.95}, \mathrm{Ca}_{0.05}\right)\left(\mathrm{Ti}_{0.96}, \mathrm{Mn}_{0.01}, \mathrm{Mg}_{0.02}, \mathrm{Fe}_{0.05}\right) \mathrm{Si}_{2.00} \mathrm{O}_{8.00} .
$$

There are two formula units in the cell, and the measured density is $4.43 \pm 0.02 \mathrm{~g} / \mathrm{cm}^{3}$. A preliminary note on the structure of fresnoite was reported earlier (Moore and Louisnathan, 1967).

## 2. Experiments

The lattice parameters of fresnoite, reported by Alfors et al. (1965), are $a=8.52 \pm 0.01 \AA$ and $c=5.210 \pm 0.001 \AA$. We prepared a powder-diffractometer pattern of fresnoite which contained
a small amount of spectrographically pure silicon as internal standard. Ni-filtered $\mathrm{Cu} \mathrm{K}_{\alpha}$ radiation was used. A least-squares fit of the corrected line positions yielded $a=8.518 \pm 0.002 \AA$ and $c=5.211 \pm 0.001 \AA$. Intensities of the $h k 0$ to $h k 5$ layers were measured on a manual scintillation-counter goniometer of Weissenberg geometry, from a single, anhedral, wedge-shaped crystal of $\sim 0.007 \mathrm{~mm}^{3}$ volume, using Zr -filtered $\mathrm{Mo} K \alpha$ radiation. The size and shape of the crystal were carefully measured, and the observed intensities were corrected for Lorentz, polarization and absorption factors.

## 3. Determination of the structure and refinement

As the only systematic extinction conditions were $0 k l, k \neq 2 n$, the space group is $P 4 / \mathrm{mbm}, P 4 b m$, or $P \overline{4} b 2$. The location of $\mathrm{Ba}-\mathrm{Ba}$ and $\mathrm{Si}-\mathrm{Si}$ peaks on the $z$-axis Patterson projection ruled out $P \overline{4} b 2$ as a possible space group. After assigning Ti to the origin in the $x y$ plane, the positions of Ba and Si were obtained by identifying the $\mathrm{Ba}-\mathrm{Ti}, \mathrm{Ba}-\mathrm{Si}$ and $\mathrm{Ti}-\mathrm{Si}$ peaks. The expected tetrahedral geometry of the silicate groups allowed approximate location of the oxygen atoms and the space group was identified as $P 4 b m$. The $z$ coordinates of $\mathrm{Ti}, \mathrm{Si}$, and three oxygen atoms were calculated from packing considerations, with Ba arbitrarily placed at $z=0$. The fourth oxygen atom was placed $2 \AA$ immediately below the Ti atom, and a structurefactor calculation showed that the model was satisfactory.

Using the least-squares SORFLS program (a local modified version of the ORFLS program of BUSING et al. (1962) the structural parameters were refined. With unit weights and one scale factor for all reflections, and isotropic temperature factors for all atoms, six cycles gave $R=0.11$. At this stage, different scale factors for each of the $l$ layers were introduced and anomalous-dispersion correction terms for the scattering factors were applied as follows:

| Atom | $f_{\theta=0^{\circ}}^{\prime}$ | $f_{\theta=0^{\circ}}^{\prime \prime}$ |
| :--- | ---: | ---: |
| Ba | -0.60 | 2.65 |
| Ti | 0.20 | 0.55 |
| Si | 0.10 | 0.10 |

It was assumed that $f^{\prime}$ and $f^{\prime \prime}$ do not depend on $\theta$. Four cycles brought $R$ to 0.095 . Next, a weighting scheme with $w=1 /\left(4.4+\left|F_{0}\right|+\left|F_{0}\right|^{2} / 260\right)$ was introduced. In order to determine the absolute configuration,
two parallel refinements, one on $h k l$ and the other on $\bar{h} \bar{k} \bar{l}$, were performed. After four cycles the results were

|  | $R$ | $R^{\prime}$ |
| :--- | :---: | :---: |
| Refinement on $h k l:$ | 0.0838 | 0.0805 |
| Refinement on $\overline{h k} \bar{k}:$ | 0.0840 | 0.0806. |

Further refinements were discontinued since subsequent iterations neither lowered $R$ nor reduced the errors in the structural parameters. The differences in the $z$ coordinates of the atoms between the two polarities, relative to the Ba atom, were within the errors. The preference of positive over negative polarity cannot be made with confidence. A possible reason for the insignificant differences between the two polarities is discussed later. Table 1 gives the values of $F_{0}(h k l)$ and $F_{\mathrm{c}}(h k l)$ as calculated in the final refinement of the positive
Table 1. Observed and calculated structure amplitudes in fresnoite The overall scale factor is $0.6 \times 10^{-2}$



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Table 2. Atomic coordinates and isotropic temperature factors of atoms in fresnoite The estimated standard deviation in these quantities are given in the parentheses (The first value is for the positive polarity and the second is for the negative polarity)

| Atom | Coordinates <br> $y$ |  |  | $z$ |
| :--- | :--- | :--- | :--- | :--- |
| Ba | $0.3272(01)$ | $\frac{1}{2}+x$ | 0 | $\beta$ |
|  | $0.3272(01)$ | $\frac{1}{2}+x$ | 0 | $0.89(06)$ |
| Si | $0.1282(06)$ | $\frac{1}{2}+x$ | $0.5207(30)$ | $0.89(06)$ |
|  | $0.1281(06)$ | $\frac{1}{2}+x$ | $0.5136(30)$ | $0.39(13)$ |
| Ti | 0 | 0 | $0.5414(18)$ | $0.17(12)$ |
|  | 0 | 0 | $0.5385(18)$ | $0.17(13)$ |
| $0(1)$ | 0 | 0.5 | $0.6183(91)$ | $1.80(72)$ |
|  | 0 | 0.5 | $0.6091(96)$ | $1.66(69)$ |
| $0(2)$ | $0.1236(19)$ | $\frac{1}{2}+x$ | $0.2125(64)$ | $0.51(37)$ |
|  | $0.1241(20)$ | $\frac{1}{2}+x$ | $0.2019(62)$ | $0.47(37)$ |
| $0(3)$ | $0.2896(23)$ | $0.5762(23)$ | $0.6584(45)$ | $1.29(33)$ |
|  | $0.2897(23)$ | $0.5758(23)$ | $0.6496(45)$ | $1.23(33)$ |
| $0(4)$ | 0 | 0 | $0.2278(91)$ | $0.94(57)$ |
|  | 0 | 0 | $0.2178(95)$ | $0.96(57)$ |

Table 3. Interatomic distances and bond angles in fresnoite
The estimated standard deviations in these quantities are given in parentheses

| Ti polyhedron |  |
| :--- | :--- |
| $\mathrm{Ti}-\mathrm{O}(3)$ | $=2.001(21) \AA$ |
| $\mathrm{Ti}-\mathrm{O}(4)$ | $=1.634(45)$ |
| $\mathrm{O}(3)-\mathrm{O}(4)$ | $=2.944(43)$ |
| $\mathrm{O}(3)-\mathrm{O}\left(3^{\prime}\right)$ | $=2.696(28)$ |
| $\mathrm{O}(4)-\mathrm{Ti}-\mathrm{O}(3)$ | $=107.74^{\circ}(34)$ |
| $\mathrm{O}(3)-\mathrm{Ti}-\mathrm{O}\left(3^{\prime}\right)$ | $=144.54 \quad(61)$ |
| $\mathrm{Si}-\mathrm{O}(3)-\mathrm{Ti}$ | $=135.82 \quad(47)$ |

Ba polyhedron
$\mathrm{Ba}-\mathrm{O}(3)[2 \times]=2.801(21)$
$\mathrm{Ba}-\mathrm{O}(2) \quad=2.690(41)$
$\mathrm{Ba}-\mathrm{O}\left(3^{\prime \prime \prime}\right)[2 \times]=3.157$ (34)
$\mathrm{Ba}-\mathrm{O}\left(2^{\prime}\right)[2 \times]=2.788(30)$
$\mathrm{Ba}-\mathrm{O}(1) \quad=2.880(36)$
$\mathrm{Ba}-\mathrm{O}(4)[2 \times]=3.368(31)$
$\mathrm{Si}_{2} \mathrm{O}_{7}$ group

| $\mathrm{Si}-\mathrm{O}(1)$ | $=1.626(37)$ |
| :--- | :--- |
| $\mathrm{Si}-\mathrm{O}(2)$ | $=1.607(39)$ |
| $\mathrm{Si}-\mathrm{O}(3)[2 \times]$ | $=1.613(25)$ |
| $\mathrm{O}(3)-\mathrm{O}\left(3^{\prime \prime}\right)$ | $=2.569(40)$ |
| $\mathrm{O}(3)-\mathrm{O}(2)[2 \times]$ | $=2.750(33)$ |
| $\mathrm{O}(3)-\mathrm{O}(1)[2 \times]$ | $=2.559(20)$ |
| $\mathrm{O}(2)-\mathrm{O}(1)$ | $=2.587(26)$ |
| $\mathrm{Si}-\mathrm{O}(1)-\mathrm{Si}^{\prime}$ | $=143.51^{\circ}(60)$ |

polarity. The final atomic coordinates, site-occupancy factors and the isotropic thermal-vibration parameters for both polarities are presented in Table 2.

Interatomic distances and bond angles together with the estimated standard errors in these quantities were calculated using the SORFFE program (a local modified version of the ORFFE program of Busing et al., 1964). Calculations of errors in bond distances and angles took into account the errors in the cell dimensions and in the atomic coordinates. The bond distances and angles presented in Table 3 do not include any corrections for the thermal vibrations of the atoms.

## 4. Discussion

Figure 1 is a projection of the fresnoite structure along the $x$ axis. The titanium atom is five-coordinated with tetragonal pyramidal geometry. These tetragonal pyramids occupy the corners and the base center of the primitive tetragonal cell. All the basal oxygen atoms of the Ti-O pyramid are shared by the adjacent Si tetrahedra, and the $\mathrm{Si}-\mathrm{O}$ tetrahedra are joined in pairs forming $\mathrm{Si}_{2} \mathrm{O}_{7}$ groups. The Ti-O pyramid and these groups are linked into heterocyclic fivemembered rings parallel to (001) forming sheets. Adjacent sheets are held together by $\mathrm{Ba}-\mathrm{O}$ bonds. Thus, the crystal structure of fresnoite is markedly similar to that of melilite.


Fig. 1. Crystal structure of fresnoite; $x$-axis projection
a) The Ti-O tetragonal pyramid

The crystal chemistry of most transition metals is dominated by tetrahedral and octahedral coordination, and five-coordination is relatively unusual, especially in naturally occurring minerals. The trigonal-bipyramidal configuration of oxygen atoms around Al in andalusite and Al and Mg in grandidierite are two of the rare examples of five-coordination in minerals. Kimball (1940) proposed the valencebond theory of directed hybrid orbitals and showed that $d s p^{3}$ or $d^{3} s p$ hybrids which use the $d_{z^{2}}$ atomic orbital would give rise to trigonalbipyramidal configuration and that the $d^{2} s p^{2}, d^{4} s, d^{2} p^{3}$ or $d^{4} p$ hybrids which use the $d_{x^{2}-y^{2}}$ atomic orbital would give rise to a tetragonalpyramidal configuration. Very few instances of five-coordination were known before the early 1960's. In the past decade approximately forty complex organometallic compounds have been discovered whose metal atom is five-coordinated in tetragonal-pyramidal configuration. With the increasing knowledge of the geometry of these compounds it became clear that the valence-bond theory of directed orbitals alone could not satisfactorily explain the observed variations in the dimensions of the tetragonal pyramid. Moreover, the theory offered no reasonable means of predicting whether a central atom would employ its $d_{z^{2}}$ orbital or the $d_{x^{2}-y^{2}}$ orbital. Zemann (1963) and Gillespie (1963), working individually, published satisfactory theories based on a point-charge model, considering the steric effect resulting from the electrostatic repulsive forces. Their theories, aside from mutually agreeing, provide a better explanation for the observed bond lengths and bond angles. They found that the trigonal bipyramid is energetically the most favorable configuration, very closely followed by a tetragonal pyramid with the apex-central atom-basal atom angle (called the apical angle at the central atom) of $104^{\circ} 04^{\prime}$. Zemann (1963) found that the energy difference between the trigonal bipyramid and the tetragonal pyramid for an $\mathrm{AB}_{5}^{2-}$ complex is only 0.258 eV/complex and suggests that transitions between the two states are very likely.

The dimensions of the Ti tetragonal pyramid are given in Table 3. The Ti-O ionic-bond distance, calculated using the ionic radii of Zachariasen (unpublished), is $2.06 \AA$. An approximate estimate of the $\mathrm{Ti}-\mathrm{O}$ single-bond covalent distance, as calculated using Pauling's (1960) single-bond distances, is $1.98 \AA$. The four basal oxygen atoms are at a distance of $2.00 \AA$ and the apical oxygen is at a distance of $1.63 \AA$ from
the Ti atom, indicating a high degree of covalency in the tetragonal pyramid. Indeed, the electrostatic-valence bond-strength calculation, $\zeta$, is $\frac{4}{5}+\frac{2}{10}+\frac{2}{10}+\frac{2}{10}+\frac{2}{10}=1.60$, indicating extreme cation undersaturation in the apical oxygen. The apical angle at Ti, namely, the $\mathrm{O}(4)-\mathrm{Ti}-\mathrm{O}(3)$ angle, is $107^{\circ} 42^{\prime}$, considerably larger than the predicted angle of $104^{\circ} 04^{\prime}$ (Zemann, 1963). The basal oxygen atoms, $\mathrm{O}(3)$, have a nearest cation-neighbor coordination number of four, suggesting a state of partial nonequivalent-sp ${ }^{3}$ hybridization. One of the four hybrids is used in $\sigma$ bonding to Si , another in $\sigma$ bonding to Ti and the remaining two enhance the $\pi$ overlap both to Ti and Si . Thus, the $\mathrm{Ti}-\mathrm{O}(3)$ bonds could be expected to have bond orders between 1 and $1^{1 / 2}$. The apical oxygen, $\mathrm{O}(4)$, has one nearest neighbor of Ti , and four second-nearest neighbors of Ba at such large distances that the latter bonds are almost purely ionic in character. Thus the apical oxygen is not in any hybrid state, and it can donate an electron to the Ti atom enabling the latter to form a set of five hybrid orbitals which in turn take part in double to triple bonds to the apical oxygen, giving a very short $\mathrm{Ti}-\mathrm{O}(4)$ distance. If repulsions between the bonding electrons are considered, the apical angle at Ti would be expected to increase to a larger value than the expected value of $104^{\circ} 04^{\prime}$, as observed in fresnoite. Lamprophyllite is the only other titanosilicate where Ti has oxygen ligands in tetragonal-pyramidal geometry (Woodrow, 1964). Unfortunately, the dimensions of the tetragonal pyramid in this crystal are not accurately known. In oxo-bis-acetyl-acetone-vanadium (IV), vanadium is in a tetragonal pyramid of five oxygen atoms. This vanadium complex crystallizes in space group $P \overline{1}$, the $V-O$ distances to the basal oxygen atoms being $1.96,1.96,1.97$ and $1.98 \AA$, with an apical oxygen distance of $1.56 \AA$, and the apical angle at V is $106^{\circ}$ (Dodge, Templeton and Zalkin, 1961). The crystal structure of the mineral haradaite, $\mathrm{Sr}(\mathrm{VO})\left[\mathrm{Si}_{2} \mathrm{O}_{6}\right]$ (Takeuchi and Joswig, 1967) has similar values: four V - O 1.99 , one $\mathrm{V}-\mathrm{O} 1.57 \AA$.

Masse, Grenier and Durif (1967) have published a description of the crystal structure of fresnoite when our preliminary report was in the press. Although there is no discrepancy in the gross features of the structure, some of their interatomic distances are questionable, especially their $\mathrm{Ti}-\mathrm{O}(4)$ bond length of $1.98 \AA$ and some of the dimensions of the $\mathrm{Si}_{2} \mathrm{O}_{7}$ group. The equalization of all the $\mathrm{Ti}-\mathrm{O}$ distances is certainly invalid. The discrepancies in the $\mathrm{Si}_{2} \mathrm{O}_{7}$ group will be considered shortly.
b) The $\mathrm{Si}_{2} \mathrm{O}_{7}$ group

The general theme of chemical bonding within the $\mathrm{Si}_{2} \mathrm{O}_{7}$ group in fresnoite is identical to that observed in melilites; quantitatively, as inferred from the bond lengths and bond angles, there are significant differences. In discussing the nature of bonding within the $\mathrm{Si}_{2} \mathrm{O}_{7}$ group in Zn melilite (Louisnathan, 1969) it was predicted that the dimensions of the pyrosilicate group would vary with different substitutions in the Zn or Ca sites. With the increase in the covalency of the bonding between oxygen atoms and the cations other than silicon, there also will be an increasing degree of equalization of bond lengths between the bridge-bonds and the terminal-bonds. The $\mathrm{Ti}, \mathrm{Si}$ bridging oxygen, $O(3)$, shares two of its four nonequivalent-sp ${ }^{3}$ hybrid orbitals in the bonding between Ti and Si . The unshared oxygen, $\mathrm{O}(2)$, is again more covalently bonded to Ba in fresnoite, than it is to Ca in hardystonite. The effect of this is to slightly lengthen the $\mathrm{Si}-\mathrm{O}(2)$ bond relative to that oberved in hardystonite. The increase in the $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angle in fresnoite relative to that observed in hardystonite is again due to an increased participation of the bridging oxygen in covalent bonding to a cation other than Si . The dimensions of the $\mathrm{Si}_{2} \mathrm{O}_{7}$ group observed in fresnoite and in hardystonite are compared in Table 4; the variations closely follow the above arguments. Table 4 also

Table 4. Comparison of the dimensions of $\left[\mathrm{Si}_{2} \mathrm{O}_{7}\right]$ group in fresnoite and hardystonite

|  | Fresnoite | Hardystonite | Fresnoite <br> (MASSE et al., 1967) |
| :--- | :--- | :--- | :--- |
| $\mathrm{Si}-\mathrm{O}(1)$ | $1.626(37) \AA$ | $1.649(03) \AA$ | $1.65 \AA$ |
| $\mathrm{Si}-\mathrm{O}(2)$ | $1.607(39)$ | $1.583(01)$ | 1.62 |
| $\mathrm{Si}-\mathrm{O}(3)$ | $1.613(25)[2 \times]$ | $1.619(04)[2 \times]$ | 1.60 |
| $\mathrm{Si}-\mathrm{O}(1)-\mathrm{Si}^{\prime}$ angle | $143.51^{\circ}(60)$ | $138.52^{\circ}(33)$ | $143^{\circ}$ |

presents the dimensions of the $\mathrm{Si}_{2} \mathrm{O}_{7}$ group obtained by Masse et al. (1967); their dimensions nearly correspond to those which would be found in an isolated pyrosilicate group, which is not true in this structure. Probably, the discrepancies observed in the various bond parameters between this work and that of Masse et al. (1967) are largely due to insufficient data ( 50 reflections of $h k 0$ and $0 k l$ type) in the latter work, and lack of full three-dimensional refinement.
c) Ba polyhedron

Barium in fresnoite is 10 -coordinated. The expected coordination number when Ba cation is surrounded by oxygen anions is 12 , and the $\mathrm{Ba}-\mathrm{O}$ ionic-bond distance, as calculated using Zachariasen's (unpublished) ionic radii, is $2.94 \AA$. In the different Ba perovskites, where Ba is in 12 -coordination, the $\mathrm{Ba}-\mathrm{O}$ distance varies from 2.95 to $3.11 \AA$. The $\mathrm{Ba}-\mathrm{O}$ distances in fresnoite are given in Table 3. There are six distances between 2.69 to $2.88 \AA$ that are significantly shorter than the ionic-bond length, indicating the presence of considerable covalent character in these bonds. The Ba polyhedron is


Fig.2. The $\mathrm{BaO}_{10}$ polyhedron at point symmetry $m$. The dashed lines are bonds that reach upward from Ba atom and the dotted lines are bonds that reach downward
a slightly distorted pentagonal antiprism, the distortion being mainly in the areas of the top and bottom faces of the antiprism. Though the true symmetry of the Ba polyhedron is $C_{s}$, an approximate $S_{10}$ symmetry results since there exist a nearly one-to-one correspondence between the bonds that reach up and down as shown in Fig. 2. The effect of this on the outer electrons of barium atom would be to somewhat make them spherically symmetrical, and, in turn, the effect of this on the anomalous scattering of $x$-rays by barium is to nearly equalize $F(h k l)$ and $F(\bar{h} \bar{k} \bar{l})$. This is a possible reason for the nonresolution of the two polarities during the refinement of the structure.

## d) Thermal-vibration parameters

The nature of chemical bonding discussed above leads to the following predictions regarding the thermal motion of the atoms in
fresnoite. With 1 to $1 \frac{1}{2}$ bond orders reaching Ti from basal oxygen atoms, and 2 to 3 bonds from the apical oxygen atom, the titanium is the most rigidly bonded atom in the structure, followed by Si and then by Ba. $\beta_{\mathrm{Ba}}=0.89>\beta_{\mathrm{Si}}=0.37>\beta_{\mathrm{Ti}}=0.17$ is thus in conformity with the arguments given. Similarly, for the anions, the predicted order of increasing $\beta$ is $\mathrm{O}(2), \mathrm{O}(4), \mathrm{O}(3)$ and $\mathrm{O}(1)$, as observed.

## 5. Relationship of crystal structure to paragenesis

Fresnoite is associated with sanbornite, gillespite and a number of other new minerals described by Alfors et al. (1965). The assemblage is typically a high-temperature type as inferred from the melting points of the minerals, and fresnoite has a high melting point relative to a number of other minerals with which it is associated. Geologically the occurrence of fresnoite is in the migmatite zone. The petrology of this area is described by Macdonald (1941). The observed average grain size of about 0.3 mm , contrasted with the late-stage giant crystals of sanbornite, and the high melting point suggest that fresnoite was one of the early products to crystallize out of the migmatic fluid, crystallizing almost immediately after coming into contact with the walls of the host rocks. Вовоvich (1963), and Bobovich and Petrovskil (1963) have made a spectroscopic study of the coordination state of titanium in glassy and recrystallized silicate melts, and have found that titanium has 4 and 5 coordination in glasses which, on recrystallization, passes on to 6 coordination. Thus it is possible that the five coordination in fresnoite reflects the genesis of the mineral.

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